Information and Optimal Trading Strategies with Dark Pools

Internet Appendices

February 2021

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Internet Appendix I (Expected Profits for Each Type of Order)

In this Appendix we explain how the expected profits of each possible type of order are calculated. Note that the expected profits for each rational trader depend on the information contained in the LOB and the information the trader has about the liquidation value of the asset. Note that the game is symmetric and, hence, we focus on symmetric equilibria.

Let us define Ω_o and Γ_o as the probability that an informed trader and uninformed trader at $t = 1$ choose an order $\mathcal{O} \in \mathbb{O}_D$, where $o = 0$ corresponds to a NT order; $o = 1$ to a MO; $o = 2$ to a LO; $o = 3$ to a DO; and such that $\sum_{o=0}^{3} \Omega_o = 1$ and $\sum_{o=0}^{3} \Gamma_o = 1$.

We also define as $\mathbb{B}_t$ the set of all possible states of the LOB after a trader arrived at t and eventually traded, and as $\mathcal{B}_t \in \mathbb{B}_t$ a possible state of the LOB. For example, $\mathcal{B}_1$ is a possible state of the LOB such that

$$\mathcal{B}_1 = \begin{cases} \varnothing, & \text{if the best prices in the } LOB \text{ are } (A_1^1, B_1^1), \\ BMO, & \text{if the best prices in the } LOB \text{ are } (A_1^2, B_1^1), \\ BLO, & \text{if the best prices in the } LOB \text{ are } (A_1^1, B_1^1 + \tau), \\ SMO, & \text{if the best prices in the } LOB \text{ are } (A_1^1, B_1^2), \\ SLO, & \text{if the best prices in the } LOB \text{ are } (A_1^1 - \tau, B_1^1). \end{cases}$$

We also define $\mathcal{B}_2^{End}$ a possible state of the LOB at the end of the period $t = 2$.

In the second trading period the expected profits of each strategy depends on the state of the LOB (which on its turn depends on the chosen strategy at $t = 1$). Uniformed traders at $t = 2$ form beliefs about the strategies and type of player in $t = 1$. Thus, the uninformed traders' belief at $t = 2$ about the probability that the MO, LO, and observed in the LOB was submitted by an informed trader as X, Y and Z, respectively and are defined in the paper by $(B.1)$, $(B.2)$ and $(C.1)$.

We start solving the model backwards at $t = 2$, the last date when a rational trader can decide where and how to trade. We consider at $t = 2$ all the possible states. There are 5 possible states depending on which type of trader arrived (LT, IH, IL, UB, US). Also, depending on which was the traders' optimal choice at $t = 1$, there are five possible states of the LOB at the end of $t = 1$. In what follows we calculate the expected profits in each of the possible combinations of types of trader and states of the LOB. Notice, that in what follows we do not include the profits of choosing NT since these are always null in all the possible states and times.

$t = 2$

2.1. State ($t = 2$, Liquidity trader)

A liquidity trader arrives with probability $1 - \lambda$. With probability 1/2 he will choose a BMO and with probability 1/2 a SMO.

2.2. State ($t = 2$, IH)

In this state, the liquidation value of the asset is $v = v^H$, and an informed trader arrives. This trader is a buyer. The best prices of the LOB at the end of $t = 1$ can take the following values: (A_1^1, B_1^1), (A_1^2, B_1^1), $(A_1^1, B_1^1 + \tau)$, (A_1^1, B_1^2), or $(A_1^1 - \tau, B_1^1)$. Hence, we distinguish five possible states of the LOB at the beginning of the second trading period.

2.2.1. (A_1^1, B_1^1)

This occurs when either a trader that arrives at the market at $t = 1$ chooses to go to the DP or chooses NT. Therefore, $\mathcal{B}_1 = \varnothing$. The expected profits for the various types of orders at $t = 2$ are as follows:

$$\begin{aligned} \mathbb{E}\left(\Pi_{BMO,2}^{IH} | \mathcal{B}_1 = \varnothing, v = v^H\right) &= v^H - A_1^1 = (\kappa - k_1)\,\tau > 0, \text{ and} \\ \mathbb{E}\left(\Pi_{BDO,2}^{IH} | \mathcal{B}_1 = \varnothing, v = v^H\right) &= \theta_2^I \left(v^H - \frac{A_1^1 + B_1^1}{2}\right) = \theta_2^I \kappa \tau \geq 0. \end{aligned}$$

For computing the expected profits of a BLO, note that a BLO is chosen by an informed trader who observes v^H (IH) and arrives at the market at $t = 2$ can only be executed if at $t = 1$ there is an uninformed seller who chooses a DO and the order is not executed. Furthermore, given that the trader at $t = 2$ observes v^H, the possible cases such that the LOB has not changed during $t = 1$ are the following: 1) an informed trader who observes v^H and who goes to the dark at $t = 1$; 2) an uninformed buyer who goes to the dark at $t = 1$; 3) an uninformed seller who goes to the dark at $t = 1$; 4) an uninformed buyer who chooses NT at $t = 1$; and 5) an uninformed seller who chooses NT at $t = 1$.

Therefore, the probability of execution of a BLO chosen by an informed trader who observes v^H and faces (A_1^1, B_1^1), which is denoted by $p_{BLO,2}^{IH}(\mathcal{B}_1 = \varnothing)$, is given by

$$p_{BLO,2}^{IH}(\mathcal{B}_1 = \varnothing) = \frac{(1 - \theta_1^U)\frac{1-\pi}{2}\Gamma_3}{\pi\Omega_3 + (1 - \pi)(\Gamma_0 + \Gamma_3)},$$

and the corresponding expected profits are given by

$$\mathbb{E}\left(\Pi_{BLO,2}^{IH} | \mathcal{B}_1 = \varnothing, v = v^H\right) = p_{BLO,2}^{IH}(\mathcal{B}_1 = \varnothing)\,\delta(v^H - B_1^1 - \tau) = p_{BLO,2}^{IH}(\mathcal{B}_1 = \varnothing)\,\delta\,(\kappa + k_1 - 1)\,\tau.$$

Comparing profits, we conclude that the informed trader never chooses NT since this option is strictly dominated by a MO.

2.2.2. (A_1^2, B_1^1)

This state of the LOB occurs when at $t = 1$ a trader arrives at the market and chooses a BMO (denoted by $\mathcal{B}_1 = BMO$). In this case the LOB has changed in the ask side: $A_2^1 = A_1^2$ and $B_2^1 = B_1^1$.

The expected profits for the various types of orders at $t = 2$ are as follows:

$$\mathbb{E}\left(\Pi^{IH}_{BMO,2}|\mathcal{B}_1 = BMO, v = v^H\right) = v^H - A_1^2 = (\kappa - k_2)\,\tau > 0,$$
$$\mathbb{E}\left(\Pi^{IH}_{BDO,2}|\mathcal{B}_1 = BMO, v = v^H\right) = \theta_2^I\left(v^H - \frac{A_1^2 + B_1^1}{2}\right) = \theta_2^I\left(\kappa - \frac{k_2 - k_1}{2}\right)\tau \geq 0, \text{ and}$$

$$\mathbb{E}\left(\Pi^{IH}_{BLO,2}|\mathcal{B}_1 = BMO, v = v^H\right) = 0,$$

since the probability of execution of any LO at $t = 2$ is null. Comparing expected profits, we conclude that the informed trader does not choose a LO or NT since these options are strictly dominated by a MO.

2.2.3. $(A_1^1, B_1^1 + \tau)$

Note that the LOB has changed in the bid side because during the first trading period a trader has arrived at the market and has chosen a BLO (denoted by $\mathcal{B}_1 = BLO$). The expected profits for the various types of orders at $t = 2$ are as follows:

$$\mathbb{E}\left(\Pi^{IH}_{BMO,2}|\mathcal{B}_1 = BLO, v = v^H\right) = v^H - A_1^1 = (\kappa - k_1)\,\tau > 0,$$
$$\mathbb{E}\left(\Pi^{IH}_{BDO,2}|\mathcal{B}_1 = BLO, v = v^H\right) = \theta_2^I\left(v^H - \frac{A_1^1 + B_1^1 + \tau}{2}\right) = \theta_2^I\left(\kappa - \frac{1}{2}\right)\tau \geq 0, \text{ and}$$
$$\mathbb{E}\left(\Pi^{IH}_{BLO,2}|\mathcal{B}_1 = BLO, v = v^H\right) = 0,$$

since the probability of execution of any LO at $t = 2$ is null. Hence, the trader will not choose a LO or NT since these options are strictly dominated by a MO.

2.2.4. (A_1^1, B_1^2)

Note that during the first trading period a trader has arrived at the market and has chosen a SMO (denoted by $\mathcal{B}_1 = SMO$). The expected profits for the various types of orders at $t = 2$ are as follows:

$$\mathbb{E}\left(\Pi^{IH}_{BMO,2}|\mathcal{B}_1 = SMO, v = v^H\right) = v^H - A_1^1 = (\kappa - k_1)\,\tau > 0,$$
$$\mathbb{E}\left(\Pi^{IH}_{BDO,2}|\mathcal{B}_1 = SMO, v = v^H\right) = \theta_2^I\left(v^H - \frac{A_1^1 + B_1^2}{2}\right) = \theta_2^I\left(\kappa + \frac{k_2 - k_1}{2}\right)\tau \geq 0, \text{ and}$$
$$\mathbb{E}\left(\Pi^{IH}_{BLO,2}|\mathcal{B}_1 = SMO, v = v^H\right) = 0,$$

since the probability of execution of any LO at $t = 2$ is null. Hence, the trader will not choose a LO or NT because these options are strictly dominated by a MO.

2.2.5. $(A_1^1 - \tau, B_1^1)$

Note that during the first trading period a trader has arrived at the market and has chosen a SLO which improved the ask price (denoted by $\mathcal{B}_1 = SLO$). The expected profits for the various types of orders at $t = 2$ are as follows:

$$\begin{aligned}\mathbb{E}\left(\Pi^{IH}_{BMO,2}|\mathcal{B}_1 = SLO, v = v^H\right) &= v^H - A^1_1 + \tau = (\kappa - k_1 + 1)\,\tau > 0,\\ \mathbb{E}\left(\Pi^{IH}_{BDO,2}|\mathcal{B}_1 = SLO, v = v^H\right) &= \theta^I_2\left(v^H - \frac{A^1_1 - \tau + B^1_1}{2}\right) = \theta^I_2\left(\kappa + \frac{1}{2}\right)\tau \geq 0, \text{ and}\\ \mathbb{E}\left(\Pi^{IH}_{BLO,2}|\mathcal{B}_1 = SLO, v = v^H\right) &= 0,\end{aligned}$$

since the probability of execution of any LO at $t = 2$ is null. Hence, the trader will not choose a LO or NT since these options are strictly dominated by a MO.

2.3. State ($t = 2$, IL)

Note that the expected profits for an informed trader when $v = v^L$ (IL) are similar to the expected profits of an IH trader when $v = v^H$ since there exists the following symmetry. When the state of the LOB is (A^1_1, B^1_1), an IL and an IH always make the same choice (apart from the direction of the order). When the state of the LOB is (A^2_1, B^1_1), an IL chooses the same type of order as an IH when the state of the LOB is (A^1_1, B^2_1). When the state of the LOB is $(A^1_1, B^1_1 + \tau)$, an IL chooses the same type of order as an IH when the state of the LOB is $(A^1_1 - \tau, B^1_1)$. When the state of the LOB is (A^1_1, B^2_1), an IL chooses the same type of order as an IH when the state of the LOB is (A^2_1, B^1_1). When the state of the LOB is $(A^1_1 - \tau, B^2_1)$, an IL chooses the same type of order as an IH when the state of the LOB is $(A^1_1, B^1_1 + \tau)$.

2.4. State ($t = 2$, UB)

The best prices of the LOB at the beginning of the second trading period can take the following values: (A^1_1, B^1_1), (A^2_1, B^1_1), $(A^1_1, B^1_1 + \tau)$, (A^1_1, B^2_1), or $(A^1_1 - \tau, B^1_1)$.

2.4.1. (A^1_1, B^1_1)

This occurs when either when a trader arriving at the market at $t = 1$ chooses to go to the DP or chooses NT. Therefore, $\mathcal{B}_1 = \varnothing$.

Note that the possible cases such that the prices of the LOB have not changed during the first trading period are the following: 1) an informed trader who observes v^H and who goes to the dark at $t = 1$; 2) an informed trader who observes v^L and who goes to the dark at $t = 1$; 3) an uninformed buyer who goes to the dark at $t = 1$; 4) an uninformed seller who goes to the dark at $t = 1$; 5) an uninformed buyer who selects NT at $t = 1$; and 6) an uninformed seller who selects NT at $t = 1$. Hence,

$$\mathbb{E}(\widetilde{v}|\mathcal{B}_1 = \varnothing) = \frac{\frac{\pi}{2}\Omega_3 v^H + \frac{\pi}{2}\Omega_3 v^L + (1-\pi)(\Gamma_0 + \Gamma_3)\mu}{\pi\Omega_3 + (1-\pi)(\Gamma_0 + \Gamma_3)} = \mu,$$

and because we focus on symmetric equilibria this state of the LOB does not gives any information to the uninformed trader. The expected profits for the various types of orders at $t = 2$ are as follows:

$$\begin{aligned}
\mathbb{E}\left(\Pi^{UB}_{BMO,2}|\mathcal{B}_1=\varnothing\right) &= \mathbb{E}(\widetilde{v}|\mathcal{B}_1=\varnothing)-A^1_1=(\mu-\mu-k_1\tau)=-k_1\tau<0, \text{ and}\\
\mathbb{E}\left(\Pi^{UB}_{BDO,2}|\mathcal{B}_1=\varnothing\right) &= \theta^U_2\left(\mathbb{E}(\widetilde{v}|\mathcal{B}_1=\varnothing)-\frac{A^1_1+B^1_1}{2}\right)=\theta^U_2\kappa\left(\mu-\mu\right)\tau=0.
\end{aligned}$$

For the expected profits of a LO, note that a BLO chosen by an uniformed trader who arrives at the market at $t=2$ can only be executed if at $t=1$ there is a seller who chooses a DO and the order is not executed. Therefore, the probability of execution of a BLO chosen by an uninformed trader that faces (A^1_1,B^1_1), which is denoted by $p^{UB}_{BLO,2}\left(\mathcal{B}_1=\varnothing\right)=p\left(\mathcal{B}^{End}_2=SMO|\mathcal{B}_1=\varnothing\right)$, is given by

$$p^{UB}_{BLO,2}\left(\mathcal{B}_1=\varnothing\right)=\frac{(1-\theta^I_1)\frac{\pi}{2}\Omega_3+(1-\theta^U_1)\frac{1-\pi}{2}\Gamma_3}{\pi\Omega_3+(1-\pi)\left(\Gamma_0+\Gamma_3\right)},$$

and

$$\begin{aligned}
\mathbb{E}\left(\Pi^{UB}_{BLO,2}|\mathcal{B}_1=\varnothing\right) &= p^{UB}_{BLO,2}\left(\mathcal{B}_1=\varnothing\right)\delta(\mathbb{E}(\widetilde{v}|\mathcal{B}_1=\varnothing,\mathcal{B}^{End}_2=SMO)-B^1_1-\tau)\\
&= p^{UB}_{BLO,2}\left(\mathcal{B}_1=\varnothing\right)\delta\left(k_1-Z\kappa-1\right)\tau,
\end{aligned}$$

since

$$\begin{aligned}
\mathbb{E}(\widetilde{v}|\mathcal{B}_1 &= \varnothing,\mathcal{B}^{End}_2=SMO)=\frac{(1-\theta^U_1)\left(1-\pi\right)\Gamma_3}{(1-\theta^I_1)\pi\Omega_3+(1-\theta^U_1)\left(1-\pi\right)\Gamma_3}\mu+\\
&\quad+\frac{(1-\theta^I_1)\pi\Omega_3}{(1-\theta^I_1)\pi\Omega_3+(1-\theta^U_1)\left(1-\pi\right)\Gamma_3}v^L\\
&= \mu-Z\kappa\tau.
\end{aligned}$$

In this case the UB never chooses a MO.

2.4.2. (A^2_1,B^1_1)

This occurs when at $t=1$ a trader arrives at the market and chooses a BMO (denoted by $\mathcal{B}_1=BMO$). In this case the LOB has changed in the ask side: $A^1_2=A^2_1$ and $B^1_2=B^1_1$. Note that

$$\mathbb{E}(\widetilde{v}|\mathcal{B}_1=BMO)=\mu+\frac{\lambda\pi\Omega_1}{1-\lambda+\lambda\pi\Omega_1+\lambda\left(1-\pi\right)\Gamma_1}\kappa\tau=\mu+X\kappa\tau.$$

The expected profits for the various types of orders at $t=2$ are as follows:

$$\begin{aligned}
\mathbb{E}\left(\Pi^{UB}_{BMO,2}|\mathcal{B}_1=BMO\right) &= \mathbb{E}(\widetilde{v}|\mathcal{B}_1=BMO)-A^2_1=\left(X\kappa-k_2\right)\tau,\\
\mathbb{E}\left(\Pi^{UB}_{BDO,2}|\mathcal{B}_1=BMO\right) &= \theta^U_2\left(\mathbb{E}(\widetilde{v}|\mathcal{B}_1=BMO)-\frac{A^2_1+B^1_1}{2}\right)=\theta^U_2\left(X\kappa-\frac{k_2-k_1}{2}\right)\tau, \text{ and}\\
\mathbb{E}\left(\Pi^{UB}_{BLO,2}|\mathcal{B}_1=BMO\right) &= 0,
\end{aligned}$$

since the probability of execution of any LO is null.

2.4.3. $(A_1^1, B_1^1 + \tau)$

Note that the LOB has changed in the bid side because at $t = 1$ a trader has arrived at the market and has chosen an improving BLO (denoted by $\mathcal{B}_1 = BLO$). Note that

$$\mathbb{E}(\widetilde{v}|\mathcal{B}_1 = BLO) = \mu + \frac{\pi\Omega_2\kappa}{\pi\Omega_2 + (1-\pi)\Gamma_2}\tau = \mu + Y\kappa\tau.$$

The expected profits for the various types of orders at $t = 2$ are as follows:

$$\mathbb{E}\left(\Pi_{BMO,2}^{UB}|\mathcal{B}_1 = BLO\right) = \mathbb{E}(\widetilde{v}|\mathcal{B}_1 = BLO) - A_1^1 = (Y\kappa - k_1)\tau,$$

$$\begin{aligned}\mathbb{E}\left(\Pi_{BDO,2}^{UB}|\mathcal{B}_1 = BLO\right) &= \theta_2^U\left(\mathbb{E}(\widetilde{v}|\mathcal{B}_1 = BLO) - \frac{A_1^1 + B_1^1 + \tau}{2}\right) = \theta_2^U\left(Y\kappa - \frac{1}{2}\right)\tau, \text{ and}\\ \mathbb{E}\left(\Pi_{BLO,2}^{UB}|\mathcal{B}_1 = BLO\right) &= 0.\end{aligned}$$

2.4.4. (A_1^1, B_1^2)

This occurs when at $t = 1$ a trader arrives at the market and chooses a SMO (i.e., $\mathcal{B}_1 = SMO$). Note that

$$\mathbb{E}(\widetilde{v}|\mathcal{B}_1 = SMO) = \frac{1-\lambda+\lambda(1-\pi)\Gamma_1}{1-\lambda+\lambda(1-\pi)\Gamma_1+\lambda\pi\Omega_1}\mu + \frac{\lambda\pi\Omega_1}{1-\lambda+\lambda(1-\pi)\Gamma_1+\lambda\pi\Omega_1}v^L = \mu - X\kappa\tau.$$

The expected profits for the various types of orders at $t = 2$ are as follows:

$$\begin{aligned}\mathbb{E}\left(\Pi_{BMO,2}^{UB}|\mathcal{B}_1 = SMO\right) &= \mathbb{E}\left(\widetilde{v}|\mathcal{B}_1 = SMO\right) - A_1^1 = -\left(X\kappa + k_1\right)\tau < 0,\\ \mathbb{E}\left(\Pi_{BDO,2}^{UB}|\mathcal{B}_1 = SMO\right) &= \theta_2^U\left(\mathbb{E}\left(\widetilde{v}|\mathcal{B}_1 = SMO\right) - \frac{A_1^1 + B_1^2}{2}\right) = -\theta_2^U\left(X\kappa - \frac{k_2 - k_1}{2}\right)\tau, \text{ and}\\ \mathbb{E}\left(\Pi_{BLO,2}^{UB}|\mathcal{B}_1 = SMO\right) &= 0.\end{aligned}$$

2.4.5. $(A_1^1 - \tau, B_1^1)$

Note that at $t = 1$ a trader has arrived at the market and has chosen an improving SLO (i.e., $\mathcal{B}_1 = SLO$). Note that

$$\mathbb{E}\left(\widetilde{v}|\mathcal{B}_1 = SLO\right) = \frac{(1-\pi)\Gamma_2}{\pi\Omega_2 + (1-\pi)\Gamma_2}\mu + \frac{\pi\Omega_2}{\pi\Omega_2 + (1-\pi)\Gamma_2}v^L = \mu - Y\kappa\tau.$$

The expected profits for the various types of orders at $t = 2$ are as follows:

$$\begin{aligned}\mathbb{E}\left(\Pi_{BMO,2}^{UB}|\mathcal{B}_1 = SLO\right) &= \mathbb{E}\left(\widetilde{v}|\mathcal{B}_1 = SLO\right) - A_1^1 + \tau = -\left(Y\kappa + k_1 - 1\right)\tau \leq 0,\\ \mathbb{E}\left(\Pi_{BDO,2}^{UB}|\mathcal{B}_1 = SLO\right) &= \theta_2^U\left(\mathbb{E}\left(\widetilde{v}|\mathcal{B}_1 = SLO\right) - \frac{A_1^1 - \tau + B_1^1}{2}\right) = -\theta_2^U\left(Y\kappa - \frac{1}{2}\right)\tau, \text{ and}\\ \mathbb{E}\left(\Pi_{BLO,2}^{UB}|\mathcal{B}_1 = SLO\right) &= 0.\end{aligned}$$

In this case the UB chooses neither a MO nor a LO.

2.5. State ($t = 2$, US)

Idem sub-section 2.3.

$t = 1$

There are 5 possible states depending on which type of trader arrived (LT, IH, IL, UB, US).

1.1. State ($t = 1$, Liquidity trader)

In this case a liquidity trader arrives. Its probability is $1-\lambda$. He will choose a BMO with probability 1/2 and a SMO with probability 1/2.

1.2. State ($t = 1$, IH)

The LOB starts from its original best prices (A_1^1, B_1^1). The expected profits for the various types of orders at $t = 1$ are as follows:

$$\mathbb{E}\left(\Pi_{BMO,1}^{IH} | v = v^H\right) = v^H - A_1^1 = (\kappa - k_1)\,\tau > 0.$$

For the expected profits of a LO, note that a BLO chosen by an IH who arrives at $t = 1$ is executed only if at $t = 2$ there is a trader who chooses a SMO. However, as a US observing a BLO will never choose a MO, we conclude that the probability of execution of the LO is $(1-\lambda)/2$. Hence,

$$\mathbb{E}\left(\Pi_{BLO,1}^{IH} | v = v^H\right) = \frac{\delta\,(1-\lambda)}{2}\left(v^H - B_1^1 - \tau\right) = \frac{\delta\,(1-\lambda)}{2}\,(\kappa + k_1 - 1)\,\tau.$$

For the expected profits of a DO, note that there are two cases: 1) the order is executed in the DP and 2) the order is not executed in the DP and, then, either it is cancelled or it returns at the exchange at the end of the second trading period as a BMO.[1] In such a case, there are three possible ask prices, which depend on what has happened at $t = 2$:

1) $A_1^1 - \tau$: This ask price occurs if the trader at $t = 2$ chooses a SLO (which can only be chosen by an uninformed seller).

2) A_1^2: This ask price occurs if the trader at $t = 2$ selects a BMO (which can be chosen by an informed buyer or a liquidity trader). Notice, that an uninformed buyer observing no change in the LOB will not choose a MO (see 2.4.1.).

3) A_1^1: This ask price occurs all the other times.

[1] Note that we do not consider the case that the order returns as a BLO because its probability of execution is zero.

Therefore,

$$
\begin{aligned}
\mathbb{E}\left(\Pi^{IH}_{BDO,1}|v=v^H\right) &= \theta^I_1\left(v^H-\frac{A^1_1+B^1_1}{2}\right)+(1-\theta^I_1)\max\left\{0,\delta\left[\lambda\frac{(1-\pi)}{2}I^{US,\mathcal{B}_1=\varnothing}_{SLO,2}(v^H-A^1_1+\tau)\right.\right.\\
&+\left(\lambda\pi I^{IH,\mathcal{B}_1=\varnothing}_{BMO,2}+\frac{1-\lambda}{2}\right)(v^H-A^2_1)\\
&\left.\left.+\left(1-\lambda\frac{(1-\pi)}{2}I^{US,\mathcal{B}_1=\varnothing}_{SLO,2}-\lambda\pi I^{IH,\mathcal{B}_1=\varnothing}_{BMO,2}-\frac{1-\lambda}{2}\right)(v^H-A^1_1)\right]\right\}=\theta^I_1\kappa\tau+\\
&+(1-\theta^I_1)\max\left\{0,\delta\left[\lambda\frac{(1-\pi)}{2}I^{US,\mathcal{B}_1=\varnothing}_{SLO,2}(\kappa-k_1+1)+\right.\right.\\
&+\left(\lambda\pi I^{IH,\mathcal{B}_1=\varnothing}_{BMO,2}+\frac{1-\lambda}{2}\right)(\kappa-k_2)+\\
&\left.\left.+\left(1-\lambda\frac{(1-\pi)}{2}I^{US,\mathcal{B}_1=\varnothing}_{SLO,2}-\lambda\pi I^{IH,\mathcal{B}_1=\varnothing}_{BMO,2}-\frac{1-\lambda}{2}\right)(\kappa-k_1)\right]\tau\right\},
\end{aligned}
$$

where

$$
\begin{aligned}
I^{US,\mathcal{B}_1=\varnothing}_{SLO,2} &= \begin{cases} 1, & \text{if at } t=2\text{, an } US \text{ selects a } SLO \text{ when } \mathcal{B}_1=\varnothing\\ 0, & \text{otherwise,}\end{cases}\\
&= \begin{cases} 1, & \text{if } \dfrac{(1-\theta^I_1)\frac{\pi}{2}\Omega_3+(1-\theta^U_1)\frac{1-\pi}{2}\Gamma_3}{\pi\Omega_3+(1-\pi)(\Gamma_0+\Gamma_3)}\delta\left(k_1-1-Z\kappa\right)>0\\ 0 & \text{otherwise,}\end{cases},
\end{aligned}
$$

and

$$
\begin{aligned}
I^{IH,\mathcal{B}_1=\varnothing}_{BMO,2} &= \begin{cases} 1, & \text{if at } t=2\text{, an } IH \text{ selects a } BMO \text{ when } \mathcal{B}_1=\varnothing\\ 0 & \text{otherwise,}\end{cases}\\
&= \begin{cases} 1, & \kappa-k_1\geq\max\left\{\theta^I_2\kappa,\dfrac{(1-\theta^U_1)\frac{1-\pi}{2}\Gamma_3}{\pi\Omega_3+(1-\pi)(\Gamma_0+\Gamma_3)}\delta\left(k_1+\kappa-1\right)\right\}\\ 0, & \text{otherwise.}\end{cases}
\end{aligned}
$$

Simplifying, we obtain that:

$$
\begin{aligned}
\mathbb{E}\left(\Pi^{IH}_{BDO,1}|v=v^H\right) &= \theta^I_1\kappa\tau+(1-\theta^I_1)\max\left\{0,\delta\left[\lambda\frac{(1-\pi)}{2}I^{US,\mathcal{B}_1=\varnothing}_{SLO,2}+(\kappa-k_1)\right.\right.\\
&\left.\left.-(k_2-k_1)\left(\lambda\pi I^{IH,\mathcal{B}_1=\varnothing}_{BMO,2}+\frac{1-\lambda}{2}\right)\right]\tau\right\}=\\
&\theta^I_1\kappa\tau+(1-\theta^I_1)\delta\left[\lambda\frac{(1-\pi)}{2}I^{US,\mathcal{B}_1=\varnothing}_{SLO,2}+(\kappa-k_1)\right.\\
&\left.-(k_2-k_1)\left(\lambda\pi I^{IH,\mathcal{B}_1=\varnothing}_{BMO,2}+\frac{1-\lambda}{2}\right)\right]\tau.
\end{aligned}
$$

The last equality indicates that when an informed buyer selects to go to the DP at $t=1$ and the order is not executed, it is optimal for him to choose a MO, which returns to the exchange at the end of the second trading period.

An informed trader never chooses NT as it is dominated at least by the MO.

1.3 State $(t = 1$, IL)

Due to the symmetry of the model, the expected profits of an IL trader are the same as the ones of an IH trader.

1.4. State $(t = 1$, UB)

The LOB starts from its original best prices (A_1^1, B_1^1). The expected profits for the various types of orders at $t = 1$ are as follows:

$$\mathbb{E}\left(\Pi_{BMO,1}^{UB}\right) = \mu - A_1^1 = -k_1\tau < 0.$$

For the expected profits of a LO, note that the order gets executed if the next order is SMO which can come from an informed trader at $t = 2$ that chooses a SMO or from a liquidity trader. Note that an uninformed trader upon observing a BLO in $t = 1$ will never choose a SMO. Thus,

$$\mathbb{E}\left(\Pi_{BLO,1}^{UB}\right) = p\left(\mathcal{B}_2 = SMO|\mathcal{B}_1 = BLO\right)\mathbb{E}\left(\Pi_{BLO,1}^{UB}|\mathcal{B}_1 = BLO, \mathcal{B}_2 = SMO\right),$$

where

$$p\left(\mathcal{B}_2 = SMO\right) = \frac{\lambda}{2}\pi I_{SMO,2}^{IL,\mathcal{B}_1=BLO} + \frac{1-\lambda}{2}$$

and

$$\mathbb{E}\left(\Pi_{BLO,1}^{UB}|\mathcal{B}_1 = BLO, \mathcal{B}_2 = SMO\right) = \delta\left(\mathbb{E}\left(\widetilde{v}|\mathcal{B}_1 = BLO, \mathcal{B}_2 = SMO\right) - B_1^1 - \tau\right).$$

Moreover, notice that

$$\mathbb{E}\left(\widetilde{v}|\mathcal{B}_1 = BLO, \mathcal{B}_2 = SMO\right) = \mu - \frac{\lambda\pi I_{SMO,2}^{IL,\mathcal{B}_1=BLO}}{1-\lambda+\lambda\pi I_{SMO,2}^{IL,\mathcal{B}_1=BLO}}\kappa\tau.$$

So,

$$\mathbb{E}\left(\Pi_{BLO,1}^{UB}|\mathcal{B}_1 = BLO, \mathcal{B}_2 = SMO\right) = \delta\left(k_1 - 1 - \frac{\lambda\pi I_{SMO,2}^{IL,\mathcal{B}_1=BLO}}{1-\lambda+\lambda\pi I_{SMO,2}^{IL,\mathcal{B}_1=BLO}}\kappa\right)\tau, \text{ and}$$

$$\begin{aligned}
\mathbb{E}\left(\Pi_{BLO,1}^{UB}\right) &= \left(\frac{\lambda}{2}\pi I_{SMO,2}^{IL,\mathcal{B}_1=BLO} + \frac{1-\lambda}{2}\right)\delta\left(k_1 - 1 - \frac{\lambda\pi I_{SMO,2}^{IL,\mathcal{B}_1=BLO}}{1-\lambda+\lambda\pi I_{SMO,2}^{IL,\mathcal{B}_1=BLO}}\kappa\right)\tau \\
&= \frac{\delta}{2}\left((1-\lambda)(k_1-1) - \lambda\pi I_{SMO,2}^{IL,\mathcal{B}_1=BLO}(\kappa - k_1 + 1)\right)\tau,
\end{aligned}$$

with

$$I^{IL,\mathcal{B}_1=BLO}_{SMO,2} = \begin{cases} 1, & \text{if at } t=2, \text{ an } IL \text{ selects a } SMO \text{ when } \mathcal{B}_1 = BLO, \\ 0 & \text{otherwise,} \end{cases}$$
$$= \begin{cases} 1, & \text{if } \theta^I_2 \leq \dfrac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}}, \\ 0 & \text{otherwise.} \end{cases}$$

For the expected profits if the trader chooses to go to the DP, then there are two cases: 1) the order is executed in the DP and 2) the order is not executed in the DP and, then, either it is cancelled or it returns at the exchange at the end of the second trading period as a BMO. In such a case, there are three possible ask prices, which depend on what has happened at $t = 2$.

1) $A^1_1 - \tau$: This ask price occurs if the trader at $t = 2$ chooses a SLO (which can be chosen by either an informed seller or an uninformed seller).

2) A^2_1: This ask price occurs if the trader at $t = 2$ decides a BMO (which can be chosen either by an informed buyer or a liquidity trader). Notice, that an uninformed buyer observing no change in the LOB will not choose a MO (see 2.4.1).

3) A^1_1: This ask price occurs all the other times.

$$\begin{aligned}
&\mathbb{E}\left(\Pi^{UB}_{BDO,1}\right) = \theta^U_1 \left(\mu - \frac{A^1_1 + B^1_1}{2}\right) + \\
&+ \max\left\{0, (1-\theta^U_1)\delta \left[\frac{\lambda}{2}\left(\pi I^{IL,\mathcal{B}_1=\varnothing}_{SLO,2} + (1-\pi) I^{US,\mathcal{B}_1=\varnothing}_{SLO,2}\right)\left(\mathbb{E}(\widetilde{v}|\mathcal{B}_1 = \varnothing, \mathcal{B}_2 = SLO) - A^1_1 + \tau\right) + \right.\right. \\
&\quad + \left(\frac{\lambda\pi}{2} I^{IH,\mathcal{B}_1=\varnothing}_{BMO,2} + \frac{1-\lambda}{2}\right)\left(\mathbb{E}(\widetilde{v}|\mathcal{B}_1 = \varnothing, \mathcal{B}_2 = BMO) - A^2_1\right) + \\
&\quad + \left(1 - \frac{\lambda}{2}\left(\pi I^{IL,\mathcal{B}_1=\varnothing}_{SLO,2} + (1-\pi) I^{US,\mathcal{B}_1=\varnothing}_{SLO,2}\right) - \left(\frac{\lambda\pi}{2} I^{IH,\mathcal{B}_1=\varnothing}_{BMO,2} + \frac{1-\lambda}{2}\right)\right) \times \\
&\quad \left.\left. \times \left(\mathbb{E}(\widetilde{v}|\mathcal{B}_1 = \varnothing, \mathcal{B}_2 \neq SLO, BMO) - A^1_1\right)\right]\right\},
\end{aligned}$$

where

$$I^{IL,\mathcal{B}_1=\varnothing}_{SLO,2} = \begin{cases} 1, & \text{if at } t=2, \text{ an } IL \text{ selects a } SLO \text{ when } \mathcal{B}_1 = \varnothing, \\ 0, & \text{otherwise,} \end{cases}$$

$$I^{US,\mathcal{B}_1=\varnothing}_{SLO,2} = \begin{cases} 1, & \text{if at } t=2, \text{ an } US \text{ selects a } SLO \text{ when } \mathcal{B}_1 = \varnothing, \\ 0, & \text{otherwise,} \end{cases}$$

$$I^{IH,\mathcal{B}_1=\varnothing}_{BMO,2} = \begin{cases} 1, & \text{if at } t=2, \text{ an } IH \text{ selects a } BMO \text{ when } \mathcal{B}_1 = \varnothing, \\ 0, & \text{otherwise.} \end{cases}$$

After some computations, it follows that

$$I_{SLO,2}^{IL,\mathcal{B}_1=\varnothing} = \begin{cases} 1, & \begin{array}{c}\text{if } \dfrac{\frac{1-\pi}{2}(1-\theta_1^U)\Gamma_3}{\pi\Omega_3+(1-\pi)\left(\Gamma_0+\Gamma_3\right)}\delta(k_1+\kappa-1) > \kappa - k_1, \\ \text{and } \dfrac{\frac{1-\pi}{2}(1-\theta_1^U)\Gamma_3}{\pi\Omega_3+(1-\pi)\left(\Gamma_0+\Gamma_3\right)}\delta(k_1+\kappa-1) \geq \theta_2^I\kappa,\end{array} \\ 0, & \text{otherwise,} \end{cases}$$

$$I_{SLO,2}^{US,\mathcal{B}_1=\varnothing} = \begin{cases} 1, & \text{if } \dfrac{(1-\theta_1^I)\frac{\pi}{2}\Omega_3+(1-\theta_1^U)\frac{1-\pi}{2}\Gamma_3}{\pi\Omega_3+(1-\pi)\left(\Gamma_0+\Gamma_3\right)}\delta\left(k_1-1-Z\kappa\right) > 0, \\ 0, & \text{otherwise,} \end{cases}$$

$$I_{BMO,2}^{IH,\mathcal{B}_1=\varnothing} = \begin{cases} 1, & \text{if } \kappa - k_1 \geq \max\left\{\theta_2^I\kappa, \dfrac{(1-\theta_1^U)\frac{1-\pi}{2}\Gamma_3}{\pi\Omega_3+(1-\pi)\left(\Gamma_0+\Gamma_3\right)}\delta\left(k_1+\kappa-1\right)\right\}, \\ 0, & \text{otherwise.} \end{cases}$$

Note that

$$\begin{aligned}
\mathbb{E}(\widetilde{v}|\mathcal{B}_1 &= \varnothing, \mathcal{B}_2 = SLO) = \mu - \frac{\pi I_{SLO,2}^{IL,\mathcal{B}_1=\varnothing}}{\pi I_{SLO,2}^{IL,\mathcal{B}_1=\varnothing}+(1-\pi)I_{SLO,2}^{US,\mathcal{B}_1=\varnothing}}\kappa\tau, \\
\mathbb{E}(\widetilde{v}|\mathcal{B}_1 &= \varnothing, \mathcal{B}_2 = BMO) = \mu + \frac{\lambda\pi I_{BMO,2}^{IH,\mathcal{B}_1=\varnothing}}{1-\lambda+\lambda\pi I_{BMO,2}^{IH,\mathcal{B}_1=\varnothing}}\kappa\tau, \text{ and} \\
\mathbb{E}(\widetilde{v}|\mathcal{B}_1 &= \varnothing, \mathcal{B}_2 \neq SLO, BMO) = \mu + \frac{\frac{\lambda}{2}\pi\left(I_{BLO,2}^{IH,\mathcal{B}_1=\varnothing}-I_{SMO,2}^{IL,\mathcal{B}_1=\varnothing}\right)}{1-\frac{\lambda}{2}\left(\pi I_{SLO,2}^{IL,O_1=\varnothing}+(1-\pi)I_{SLO,2}^{US,O_1=\varnothing}\right)-\left(\frac{1-\lambda}{2}+\frac{\lambda}{2}\pi I_{BMO,2}^{IH,O_1=\varnothing}\right)}\kappa\tau.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\mathbb{E}\left(\Pi_{BDO,1}^{UB}\right) &= \max\Bigg\{0, (1-\theta_1^U)\delta\Bigg[\frac{\lambda}{2}\left(\pi I_{SLO,2}^{IL,\mathcal{B}_1=\varnothing}+(1-\pi)I_{SLO,2}^{US,\mathcal{B}_1=\varnothing}\right)\times \\
&\times\left(\mu - \frac{\pi I_{SLO,2}^{IL,\mathcal{B}_1=\varnothing}}{\pi I_{SLO,2}^{IL,\mathcal{B}_1=\varnothing}+(1-\pi)I_{SLO,2}^{US,\mathcal{B}_1=\varnothing}}\kappa\tau - \mu - k_1\tau + \tau\right) + \\
&+\left(\frac{1-\lambda}{2}+\frac{\lambda}{2}\pi I_{BMO,2}^{IH,\mathcal{B}_1=\varnothing}\right)\left(\mu + \frac{\lambda\pi I_{BMO,2}^{IH,\mathcal{B}_1=\varnothing}}{1-\lambda+\lambda\pi I_{BMO,2}^{IH,\mathcal{B}_1=\varnothing}}\kappa\tau - \mu - k_2\tau\right) + \\
&+\left(1-\frac{\lambda}{2}\left(\pi I_{SLO,2}^{IL,\mathcal{B}_1=\varnothing}+(1-\pi)I_{SLO,2}^{US,\mathcal{B}_1=\varnothing}\right)-\left(\frac{1-\lambda}{2}+\frac{\lambda}{2}\pi I_{BMO,2}^{IH,\mathcal{B}_1=\varnothing}\right)\right)\times \\
&\times\left(\frac{\frac{\lambda}{2}\pi\left(I_{BLO,2}^{IH,\mathcal{B}_1=\varnothing}-I_{SMO,2}^{IL,\mathcal{B}_1=\varnothing}\right)}{1-\frac{\lambda}{2}\left(\pi I_{SLO,2}^{IL,\mathcal{B}_1=\varnothing}+(1-\pi)I_{SLO,2}^{US,\mathcal{B}_1=\varnothing}\right)-\left(\frac{1-\lambda}{2}+\frac{\lambda}{2}\pi I_{BMO,2}^{IH,\mathcal{B}_1=\varnothing}\right)}\kappa\tau - k_1\tau\right)\Bigg]\Bigg\}
\end{aligned}$$

or, equivalently,

$$\begin{aligned}\mathbb{E}\left(\Pi^{UB}_{BDO,1}\right) &= \max\left\{0,(1-\theta^U_1)\delta\left[\frac{\lambda}{2}\left(\pi I^{IL,\mathcal{B}_1=\varnothing}_{SLO,2}+(1-\pi)I^{US,\mathcal{B}_1=\varnothing}_{SLO,2}\right)\right.\right.\\ &\left.\left.-\frac{k_2}{2}\left(1-\lambda+\lambda\pi I^{IH,\mathcal{B}_1=\varnothing}_{BMO,2}\right)+k_1\left(\left(\frac{1-\lambda}{2}+\frac{\lambda}{2}\pi I^{IH,\mathcal{B}_1=\varnothing}_{BMO,2}\right)-1\right)\right]\tau\right\}\\ &= \max\left\{0,(1-\theta^U_1)\delta\left[\frac{\lambda}{2}\left(\pi I^{IL,\mathcal{B}_1=\varnothing}_{SLO,2}+(1-\pi)I^{US,\mathcal{B}_1=\varnothing}_{SLO,2}\right)\right.\right.\\ &\left.\left.+\left(\frac{1-\lambda}{2}+\frac{\lambda}{2}\pi I^{IH,\mathcal{B}_1=\varnothing}_{BMO,2}\right)(k_1-k_2)-k_1\right]\tau\right\}=0.\end{aligned}$$

The last equality indicates that when an uninformed buyer selects to go to the DP at $t=1$ and the order is not executed, it is optimal for him to choose to cancel the order. To understand why the payoff at $t=1$ of a $BDO-BMO$ for the uninformed trader is always negative, let us rewrite the corresponding payoff as

$$\theta^U_1\cdot 0+(1-\theta^U_1)\delta\left(-k_1\tau+\frac{\lambda}{2}\left(\pi I^{IL,\mathcal{B}_1=\varnothing}_{SLO,2}+(1-\pi)I^{US,\mathcal{B}_1=\varnothing}_{SLO,2}\right)\tau-\left(\frac{\lambda\pi}{2}I^{IH,\mathcal{B}_1=\varnothing}_{BMO,2}+\frac{1-\lambda}{2}\right)(k_2-k_1)\tau\right).$$

This is because if a BDO is executed at $t=1$, then its expected profits are zero, which occurs with probability θ^U_1. If the order is not executed at $t=1$ and returns to the market at the end of the second trading period, which occurs with probability $1-\theta^U_1$, then expected profits depend on whether the uninformed trader who returns to the exchange decides to submit a NT or MO. If the uninformed trader selects NT then the expected profits equal zero. If he submits a MO then the profit consists of three terms. The first consists of the expected profits of a BMO at $t=1$ for an UB (i.e., $-k_1\tau$). The second is the potential increase in profits due to the possibility that at $t=2$ a new trader arrives and submits a SLO leading to a better price for the uninformed buyer (given by $\frac{\lambda}{2}\left(\pi I^{IL,\mathcal{B}_1=\varnothing}_{SLO,2}+(1-\pi)I^{US,\mathcal{B}_1=\varnothing}_{SLO,2}\right)\tau$). The third is the potential decrease in profits due to the possibility that a trader at $t=2$ submits a BMO and, consequently, the MO that arrives at the end of the second trading period from the DP is executed at a worse price (given by $(-\left(\frac{\lambda}{2}\pi I^{IH,\mathcal{B}_1=\varnothing}_{BMO,2}+\frac{1-\lambda}{2}\right)(k_2-k_1)\tau)$. We find that the increase in expected profits due to the potential arrival of a SLO at $t=2$ is not greater than $-k_1\tau$ and these losses might be even greater in case that a BMO is submitted at $t=2$. Consequently, the payoff at $t=1$ of the $BDO-BMO$ for the uninformed buyer is always negative.

It is important to point out that the expected profits of a DO are negative. Therefore, we conclude that an UB never goes to the DP at $t=1$.

1.5. State ($t=1$, US)

Note that the game is symmetric and, therefore, the uninformed seller and the uniformed buyer have identical expected profits.

A summary of the expected profits can be found in the paper. The expected profits for an informed buyer and seller at $t=2$ are summarized in Table *C.1* and Table *C.2* , respectively. The expected profits at $t = 1$ of an informed IH and an uniformed buyer UB are summarized in Table **??** and Table **??**, respectively.

Internet Appendix II (Proofs of Proposition 1, Lemma C.1 and Proposition 2)

In this Appendix we characterize the equilibria in the single-venue market model (Proposition 1) and in the two-venue market model (Lemma C.2 and Proposition 2). For notations see the paper and Internet Appendix I.

Proof of Proposition 1. The procedure we follow to check if a particular strategy profile constitutes a PBE is as follows:

1. Specify a strategy profile for rational traders at $t = 1$.
2. Update the beliefs of the uninformed trader at $t = 2$ using Bayes' rule at all information sets, whenever possible.
3. Given their beliefs, find the optimal response for the traders at $t = 2$.
4. Given the optimal response of traders at $t = 2$, and using Tables B.4 and B.5 in Appendix B in the paper find the optimal action for rational traders at $t = 1$.
5. Check if the optimal strategy profile for the traders at $t = 1$ coincide with the profile suggested in step 1.

We apply the procedure outlined above to check when each possible strategy profile can be an equilibrium.

Case A. Suppose that $k_1 > 1$.

$\mathcal{E}_1^{ND}$: (BMO, SMO, BLO, SLO)
First step. In this case $\Omega_0 = 0$, $\Omega_1 = 1$, $\Omega_2 = 0$, $\Gamma_0 = 0$, $\Gamma_1 = 0$, and $\Gamma_2 = 1$.
Second step. Using Bayes' rule we obtain that $X^{1,ND} = \dfrac{\lambda\pi}{1-\lambda+\lambda\pi}$ and $Y^{1,ND} = 0$.
Third step. Applying Lemma 1, we know that at $t = 2$ the optimal strategy of informed traders is to choose a MO, while the optimal strategy of the uninformed trader is as follows:

State of the book	UB	US
(A_1^1, B_1^1)	NT	NT
(A_1^2, B_1^1)	$\begin{cases} MO & \text{if } \frac{\lambda\pi}{1-\lambda+\lambda\pi}\kappa > k_2 \\ NT & \text{if } \frac{\lambda\pi}{1-\lambda+\lambda\pi}\kappa \leq k_2 \end{cases}$	NT
$(A_1^1, B_1^1+\tau)$	NT	NT
(A_1^1, B_1^2)	NT	$\begin{cases} MO & \text{if } \frac{\lambda\pi}{1-\lambda+\lambda\pi}\kappa > k_2 \\ NT & \text{if } \frac{\lambda\pi}{1-\lambda+\lambda\pi}\kappa \leq k_2 \end{cases}$
$(A_1^1-\tau, B_1^1)$	NT	NT

Table II.1: Optimal responses of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is (BMO, SMO, BLO, SLO).

Fourth step. Given the optimal response of traders at $t = 2$, we find the optimal action for all rational traders at $t = 1$.

Informed traders at $t = 1$ have no incentives to deviate from the prescribed strategy profile whenever

$$\kappa - k_1 \geq \delta \frac{1-\lambda}{2} (\kappa + k_1 - 1). \tag{II.1}$$

Uninformed traders at $t = 1$ have no incentives incentive to deviate from the prescribed strategy if and only if

$$(1-\lambda)(k_1 - 1) - \lambda\pi(\kappa - (k_1 - 1)) > 0. \tag{II.2}$$

Fifth step. Nobody at $t = 1$ has unilateral incentives to deviate from (BMO, SMO, BLO, SLO) when both conditions ($II.1$) and ($II.2$) are satisfied, and these conditions can be rewritten as

$$\sigma \geq \kappa^I_{MO-LO}\tau \text{ and } PIN < \psi^U_{LO-NT}, \tag{II.3}$$

where

$$\kappa^I_{MO-LO} \equiv \frac{\delta(k_1-1)(1-\lambda) + 2k_1}{2-\delta(1-\lambda)}, \ PIN \equiv \lambda\pi \text{ and } \psi^U_{LO-NT} \equiv \frac{(1-\lambda)(k_1-1)\tau}{\sigma-(k_1-1)\tau}.$$

Finally, we consider the moves that are in the equilibrium path and must take into account that (BMO, SMO, BLO, SLO) is the strategy profile chosen at $t = 1$. Combining Expression ($II.2$) and Table II.1 it follows that an uninformed traders always chooses NT at $t = 2$.

$\mathcal{E}_2^{ND}$: (BMO, SMO, NT, NT)
For the next three equilibria we provide a sketch of proof. A detailed proof could be provided on request by the authors.

In this case $\Omega_0 = 0$, $\Omega_1 = 1$, $\Omega_2 = 0$, $\Gamma_0 = 1$, $\Gamma_1 = 0$, and $\Gamma_2 = 0$.

Using Bayes' rule we obtain that $X^{2,ND} = \dfrac{\lambda\pi}{1-\lambda+\lambda\pi}$ and $Y^{2,ND}$ is undetermined $Y^{2,ND} \in$

$[0,1]$ (as Bayes' rule implies $Y^{2,ND} = \frac{0}{0}$).

We show that no trader at $t = 1$ has unilateral incentives to deviate from (BMO, SMO, NT, NT) if and only if

$$\sigma \geq \kappa^{I}_{MO-LO}\tau \text{ and } PIN \geq \psi^{U}_{LO-NT}.$$

In the following table we include the moves that are in the equilibrium path at $t = 2$ for an uninformed trader, taking into account the conditions that must be satisfied if the strategy profile chosen at $t = 1$ is (BMO, SMO, NT, NT).

State of the book	**UB**	**US**
(A_1^1, B_1^1)	NT	NT
(A_1^2, B_1^1)	$\begin{cases} MO & \text{if } \frac{\lambda\pi}{1-\lambda+\lambda\pi}\kappa > k_2 \\ NT & \text{if } \frac{\lambda\pi}{1-\lambda+\lambda\pi}\kappa \leq k_2 \end{cases}$	NT
(A_1^1, B_1^2)	NT	$\begin{cases} MO & \text{if } \frac{\lambda\pi}{1-\lambda+\lambda\pi}\kappa > k_2 \\ NT & \text{if } \frac{\lambda\pi}{1-\lambda+\lambda\pi}\kappa \leq k_2 \end{cases}$

Table II.2: Optimal choice of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is (BMO, SMO, NT, NT).

$\mathcal{E}_3^{ND}$: (BLO, SLO, BLO, BLO)

In this case $\Omega_0 = 0$, $\Omega_1 = 0$, $\Omega_2 = 1$, $\Gamma_0 = 0$, $\Gamma_1 = 0$, and $\Gamma_2 = 1$.

Using Bayes' rule we obtain that $X^{3,ND} = 0$ and $Y^{3,ND} = \pi$.

No trader at $t = 1$ has unilateral incentives to deviate from (BLO, SLO, BLO, SLO) if and only if

$$\sigma < \kappa^I_{MO-LO}\tau \text{ and } PIN < \psi^U_{LO-NT}. \tag{II.4}$$

In the following table we include the moves that are in the equilibrium path at $t = 2$ for an uninformed trader, taking into account the conditions that must be satisfied if (BLO, SLO, BLO, SLO) is the strategy profile chosen at $t = 1$.

State of the book	**UB**	**US**
(A_1^2, B_1^1)	NT	NT
$(A_1^1, B_1^1 + \tau)$	$\begin{cases} MO & \text{if } \pi\kappa > k_1 \\ NT & \text{if } \pi\kappa \leq k_1 \end{cases}$	NT
(A_1^1, B_1^2)	NT	NT
$(A_1^1 - \tau, B_1^1)$	NT	$\begin{cases} MO & \text{if } \pi\kappa > k_1 \\ NT & \text{if } \pi\kappa \leq k_1 \end{cases}$

Table II.3: Optimal choice of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is (BLO, SLO, BLO, SLO).

$\mathcal{E}_4^{ND}$: (BLO, SLO, NT, NT)

In this case $\Omega_0 = 0$, $\Omega_1 = 0$, $\Omega_2 = 1$, $\Gamma_0 = 1$, $\Gamma_1 = 0$, and $\Gamma_2 = 0$.

Using Bayes' rule we obtain that $X^{4,ND} = 0$ and $Y^{4,ND} = 1$.

We show that nobody at $t = 1$ has unilateral incentives to deviate from (BLO, SLO, NT, NT) if

$$\sigma < \kappa^I_{MO-LO}\tau \text{ and } PIN \geq \psi^U_{LO-NT}. \tag{II.5}$$

Table II.4 includes the moves that are in the equilibrium path at $t = 2$ for an uninformed trader, taking into account that (BLO, SLO, NT, NT) is the strategy profile chosen at $t = 1$.

State of the book	**UB**	**US**
(A_1^1, B_1^1)	NT	NT
(A_1^2, B_1^1)	NT	NT
$(A_1^1, B_1^1 + \tau)$	MO	NT
(A_1^1, B_1^2)	NT	NT
$(A_1^1 - \tau, B_1^1)$	NT	MO

Table II.4: Optimal responses of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is (BLO, SLO, NT, NT).

Case B. Note that when $k_1 = 1$ the conditions ($II.2$) and (**??**) are never satisfied and, therefore, the strategies (BMO, SMO, BLO, SLO) and (BLO, SLO, BLO, BLO) cannot be part of an equilibrium of the game. By contrast, when $k_1 = 1$, the conditions (**??**) and (**??**) are always satisfied. However the condition (**??**) is never satisfied when $k_1 = 1$ and, therefore, the strategy (BLO, SLO, NT, NT) cannot be either part of an equilibrium of the game. ■

Proof of Lemma C.1. We develop our proof for buyers and, by symmetry, the same expressions will follow for sellers.

In what follows, we do not consider the strategies of NT and $BDO - NT$ for the informed traders at $t = 1$ because their corresponding expected profits are always smaller than the expected profits of BMO and $BDO - BMO$, respectively (see Internet Appendix I). Notice that because of this, we write BDO to refer to $BDO - BMO$ for an informed buyer. Similarly, we do not consider the strategies of BMO, $BDO - BMO$, and $BDO - NT$ for the uninformed buyers at $t = 1$ because their corresponding expected profits are always smaller than or equal to the expected profits of NT.

We have defined in the paper as X as the uninformed traders' belief at $t = 2$ about the probability that the MO (observed in the LOB) was submitted by an informed trader, Y as the uninformed traders' belief at $t = 2$ about the probability that the LO (observed in the LOB) was submitted by an informed trader, and Z as the uninformed trader's belief at $t = 2$ about the probability that a DO that returns to the exchange as a MO at the end of the second trading period was submitted by an informed.In each equilibrium $\mathcal{E}_i^D$ we denote by $X^{i,D}$, $Y^{i,D}$, $Z^{i,D}$ the corresponding belief X, Y, Z

$$\begin{aligned} X^{i,D} &= \frac{\lambda\pi\Omega_1}{1-\lambda+\lambda\pi\Omega_1+\lambda\left(1-\pi\right)\Gamma_1} \\ Y^{i,D} &= \frac{\pi\Omega_2}{\pi\Omega_2+\left(1-\pi\right)\Gamma_2} \\ Z^{i,D} &= \frac{(1-\theta_1^I)\pi\Omega_3}{(1-\theta_1^I)\pi\Omega_3+(1-\theta_1^U)\left(1-\pi\right)\Gamma_3}. \end{aligned} \tag{II.6}$$

In addition, we have defined P_I as the probability of execution of a limit order placed by an informed trader at $t = 2$ when there is no change in the LOB during the first trading period as

$$P_I = p_{BLO,2}^{IH}\left(\mathcal{B}_1 = \varnothing\right) = p_{SLO,2}^{IL}\left(\mathcal{B}_1 = \varnothing\right) = \frac{(1-\theta_1^U)\frac{1-\pi}{2}\Gamma_3}{\pi\Omega_3+(1-\pi)(\Gamma_0+\Gamma_3)},$$

and P_U as the probability of execution of a limit order placed by an uninformed trader at $t = 2$ given that there are no changes in prices in the LOB during the first trading period, and equals

$$P_U = p_{BLO,2}^{UB}\left(\mathcal{B}_1 = \varnothing\right) = p_{SLO,2}^{US}\left(\mathcal{B}_1 = \varnothing\right) = \frac{1}{2}\frac{(1-\theta_1^I)\pi\Omega_3+\left(1-\theta_1^U\right)(1-\pi)\Gamma_3}{\pi\Omega_3+(1-\pi)\left(\Gamma_0+\Gamma_3\right)}.$$

Finally, we have defined the following constants:

$$\begin{aligned}
\theta_{X^{i,D}} &\equiv \frac{X^{i,D}\kappa - k_2}{X^{i,D}\kappa - \frac{k_2-k_1}{2}}, \\
\theta_{Y^{i,D}} &\equiv \frac{Y^{i,D}\kappa - k_1}{X^{i,D}\kappa - \frac{1}{2}}, \\
\underline{\theta} &\equiv \frac{\kappa - k_1}{\kappa}, \\
\overline{\theta} &\equiv \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}}.
\end{aligned} \tag{II.7}$$

$\mathcal{E}_1^D : (BMO, SMO, BLO, SLO)$

First step. In this case $\Omega_0 = 0$, $\Omega_1 = 1$, $\Omega_2 = 0$, $\Omega_3 = 0$, $\Gamma_0 = 0$, $\Gamma_1 = 0$, $\Gamma_2 = 1$, and $\Gamma_3 = 0$. Moreover, $\theta_2^I = \theta_1^I$ and $\theta_2^U = \theta_1^U$.

Second step. Using Bayes' rule

$$\begin{aligned} X^{1,D} &= \frac{\lambda\pi}{1-\lambda+\lambda\pi},\ Y^{1,D} = 0,\ Z^{1,D} = z \in [0,1], \\ p_{BLO,2}^{UB,\mathcal{B}_1=\varnothing} &= p_{SLO,2}^{US,\mathcal{B}_1=\varnothing} \in [0,1],\ \text{and}\ p_{BLO,2}^{IH,\mathcal{B}_1=\varnothing} = p_{SLO,2}^{IL,\mathcal{B}_1=\varnothing} \in [0,1]. \end{aligned}$$

Third step. Using step 2 and taking into account that $p_{BLO,2}^{UB}(\mathcal{B}_1 = \varnothing) = p_{SLO,2}^{US}(\mathcal{B}_1 = \varnothing) \in [0,1]$, at $t = 2$ the expected profits of uninformed traders are as follows:

UB	BMO	BDO	BLO	NT
(A_1^1, B_1^1)	$-k_1\tau$	0	$p_{BLO,2}^{UB,\mathcal{B}_1=\varnothing}\delta\left(k_1 - Z^{1,D}\kappa - 1\right)\tau$	0
(A_1^2, B_1^1)	$\left(X^{1,D}\kappa - k_2\right)\tau$	$\theta_2^U\left(X^{1,D}\kappa - \frac{k_2-k_1}{2}\right)\tau$	0	0
$(A_1^1, B_1^1+\tau)$	$-k_1\tau$	$-\frac{1}{2}\theta_2^U\tau$	0	0
(A_1^1, B_1^2)	$-\left(X^{1,D}\kappa + k_1\right)\tau$	$-\theta_2^U\left(X^{1,D}\kappa - \frac{k_2-k_1}{2}\right)\tau$	0	0
$(A_1^1-\tau, B_1^1)$	$-(k_1-1)\tau$	$\frac{1}{2}\theta_2^U\tau$	0	0

Table II.5: Expected profits of uninformed buyers at $t = 2$ when the strategy profile at $t = 1$ is (BMO, SMO, BLO, SLO).

US	SMO	SDO	SLO	NT
(A_1^1, B_1^1)	$-k_1\tau$	0	$p_{SLO,2}^{US,\mathcal{B}_1=\varnothing}\delta\left(k_1 - Z^{1,D}\kappa - 1\right)\tau$	0
(A_1^2, B_1^1)	$-\left(X^{1,D}\kappa + k_1\right)\tau$	$-\theta_2^U\left(X^{1,D}\kappa - \frac{k_2-k_1}{2}\right)\tau$	0	0
$(A_1^1, B_1^1+\tau)$	$-(k_1-1)\tau$	$\frac{1}{2}\theta_2^U\tau$	0	0
(A_1^1, B_1^2)	$\left(X^{1,D}\kappa - k_2\right)\tau$	$\theta_2^U\left(X^{1,D}\kappa - \frac{k_2-k_1}{2}\right)\tau$	0	0
$(A_1^1-\tau, B_1^1)$	$-k_1\tau$	$-\frac{1}{2}\theta_2^U\tau$	0	0

Table II.6: Expected profits of uninformed sellers at $t = 2$ when the strategy profile at $t = 1$ is (BMO, SMO, BLO, SLO).

Hence, the optimal strategies for the uninformed are:

Optimal Strategies of Uninformed Traders at $t = 2$		
State of the Book	**UB**	**US**
(A_1^1, B_1^1)	$\begin{cases} NT & \text{if } p_{BLO,2}^{UB,\mathcal{B}_1=\varnothing} = 0 \text{ or } Z^{1,D} \geq \frac{k_1-1}{\kappa} \\ BLO & \text{if } p_{BLO,2}^{UB,\mathcal{B}_1=\varnothing} > 0 \text{ and } Z^{1,D} < \frac{k_1-1}{\kappa} \end{cases}$	$\begin{cases} NT & \text{if } p_{BLO,2}^{UB,\mathcal{B}_1=\varnothing} = 0 \text{ or } Z^{1,D} \geq \frac{k_1-1}{\kappa} \\ SLO & \text{if } p_{BLO,2}^{UB,\mathcal{B}_1=\varnothing} > 0 \text{ and } Z^{1,D} < \frac{k_1-1}{\kappa} \end{cases}$
(A_1^2, B_1^1)	$\begin{cases} NT & \text{if } X^{1,D}\kappa \leq \frac{k_2-k_1}{2} \\ BDO & \text{if } \frac{k_2-k_1}{2} < X^{1,D}\kappa \leq k_2 \\ BDO & \text{if } k_2 < X^{1,D}\kappa \text{ and } \theta_2^U > \theta_{X^{1,D}} \\ BMO & \text{if } k_2 < X^{1,D}\kappa \text{ and } \theta_2^U \leq \theta_{X^{1,D}} \end{cases}$	$\begin{cases} SDO & \text{if } X^{1,D}\kappa < \frac{k_2-k_1}{2} \\ NT & \text{if } \frac{k_2-k_1}{2} \leq X^{1,D}\kappa \end{cases}$
$(A_1^1, B_1^1 + \tau)$	NT	SDO
(A_1^1, B_1^2)	$\begin{cases} BDO & \text{if } X^{1,D}\kappa < \frac{k_2-k_1}{2} \\ NT & \text{if } \frac{k_2-k_1}{2} \leq X^{1,D}\kappa \end{cases}$	$\begin{cases} NT & \text{if } X^{1,D}\kappa \leq \frac{k_2-k_1}{2} \\ SDO & \text{if } \frac{k_2-k_1}{2} < X^{1,D}\kappa \leq k_2 \\ SDO & \text{if } k_2 < X^{1,D}\kappa \text{ and } \theta_2^U > \theta_{X^{1,D}} \\ SMO & \text{if } k_2 < X^{1,D}\kappa \text{ and } \theta_2^U \leq \theta_{X^{1,D}} \end{cases}$
$(A_1^1 - \tau, B_1^1)$	BDO	NT

Table II.7: Optimal strategies of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is (BMO, SMO, BLO, SLO).

Using that $p_{BLO,2}^{IH,\mathcal{B}_1=\varnothing} = p_{SLO,2}^{IL,\mathcal{B}_1=\varnothing} \in [0,1]$, at $t = 2$ the informed traders expected profits are :

IH	BMO	BDO	BLO	NT
(A_1^1, B_1^1)	$(\kappa - k_1)\,\tau$	$\theta_2^I \kappa\tau$	$p_{BLO,2}^{IH,\mathcal{B}_1=\varnothing}\delta\,(\kappa + k_1 - 1)\,\tau$	0
(A_1^2, B_1^1)	$(\kappa - k_2)\,\tau$	$\theta_2^I \left(\kappa - \frac{k_2-k_1}{2}\right)\tau$	0	0
$(A_1^1, B_1^1+\tau)$	$(\kappa - k_1)\,\tau$	$\theta_2^I \left(\kappa - \frac{1}{2}\right)\tau$	0	0
(A_1^1, B_1^2)	$(\kappa - k_1)\,\tau$	$\theta_2^I \left(\kappa + \frac{k_2-k_1}{2}\right)$	0	0
$(A_1^1-\tau, B_1^1)$	$(\kappa - k_1 + 1)\,\tau$	$\theta_2^I \left(\kappa + \frac{1}{2}\right)\tau$	0	0

Table II.8: Expected profits of informed buyers at $t = 2$ when the strategy profile at $t = 1$ is (BMO, SMO, BLO, SLO)

IL	SMO	SDO	SLO	NT
(A_1^1, B_1^1)	$(\kappa - k_1)\,\tau$	$\theta_2^I \kappa\tau$	$p_{SLO,2}^{IL,\mathcal{B}_1=\varnothing}\delta\,(\kappa + k_1 - 1)\,\tau$	0
(A_1^2, B_1^1)	$(\kappa - k_1)\tau$	$\theta_2^I \left(\kappa + \frac{k_2-k_1}{2}\right)\tau$	0	0
$(A_1^1, B_1^1+\tau)$	$(\kappa - k_1 + 1)\tau$	$\theta_2^I \left(\kappa + \frac{1}{2}\right)\tau$	0	0
(A_1^1, B_1^2)	$(\kappa - k_2)\tau$	$\theta_2^I \left(\kappa + \frac{k_1-k_2}{2}\right)\tau$	0	0
$(A_1^1-\tau, B_1^1)$	$(\kappa - k_1)\tau$	$\theta_2^I \left(\kappa - \frac{1}{2}\right)\tau$	0	0

Table II.9: Expected profits of informed sellers at $t = 2$ when the strategy profile at $t = 1$ is (BMO, SMO, BLO, SLO)

Define BX, SX, BY, SY as

$$BX = \begin{cases} BMO \text{ if } p_{BLO,2}^{IH,\mathcal{B}_1=\varnothing} \leq \frac{\kappa-k_1}{\delta(\kappa+k_1-1)} \\ BLO \text{ if } p_{BLO,2}^{IH,\mathcal{B}_1=\varnothing} > \frac{\kappa-k_1}{\delta(\kappa+k_1-1)}, \end{cases}$$

$$SX = \begin{cases} SMO \text{ if } p_{SLO,2}^{IL,\mathcal{B}_1=\varnothing} \leq \frac{\kappa-k_1}{\delta(\kappa+k_1-1)} \\ SLO \text{ if } p_{SLO,2}^{IL,\mathcal{B}_1=\varnothing} > \frac{\kappa-k_1}{\delta(\kappa+k_1-1)}, \end{cases}$$

$$BY = \begin{cases} BDO \text{ if } p_{BLO,2}^{IH,\mathcal{B}_1=\varnothing} < \frac{\theta_2^I\kappa}{\delta(\kappa+k_1-1)} \\ BLO \text{ if } p_{BLO,2}^{IH,\mathcal{B}_1=\varnothing} \geq \frac{\theta_2^I\kappa}{\delta(\kappa+k_1-1)}, \end{cases}$$

$$SY = \begin{cases} SDO \text{ if } p_{BLO,2}^{IH,\mathcal{B}_1=\varnothing} < \frac{\theta_2^I\kappa}{\delta(\kappa+k_1-1)} \\ SLO \text{ if } p_{BLO,2}^{IH,\mathcal{B}_1=\varnothing} \geq \frac{\theta_2^I\kappa}{\delta(\kappa+k_1-1)}. \end{cases}$$

The optimal strategy for an informed trader at $t = 2$ is:

Condition	Optimal Strategies of Informed Traders at t =2		
	State of the Book	**IH**	**IL**
Case I_1 $\theta_2^I \leq \frac{\kappa-k_2}{\kappa-\frac{k_2-k_1}{2}}$	(A_1^1, B_1^1) (A_1^2, B_1^1) $(A_1^1, B_1^1+\tau)$ (A_1^1, B_1^2) $(A_1^1-\tau, B_1^1)$	BX BMO BMO BMO BMO	SX SMO SMO SMO SMO
Case I_2 $\frac{\kappa-k_2}{\kappa-\frac{k_2-k_1}{2}} < \theta_2^I \leq \frac{\kappa-k_1}{\kappa+\frac{k_2-k_1}{2}}$	(A_1^1, B_1^1) (A_1^2, B_1^1) $(A_1^1, B_1^1+\tau)$ (A_1^1, B_1^2) $(A_1^1-\tau, B_1^1)$	BX BDO BMO BMO BMO	SX SMO SMO SDO SMO
Case I_3 $\frac{\kappa-k_1}{\kappa+\frac{k_2-k_1}{2}} < \theta_2^I \leq \frac{\kappa-k_1}{\kappa}$	(A_1^1, B_1^1) (A_1^2, B_1^1) $(A_1^1, B_1^1+\tau)$ (A_1^1, B_1^2) $(A_1^1-\tau, B_1^1)$	BX BDO BMO BDO BMO	SX SDO SMO SDO SMO
Case I_4 $\frac{\kappa-k_1}{\kappa} < \theta_2^I \leq \frac{\kappa-k_1}{\kappa-\frac{1}{2}}$	(A_1^1, B_1^1) (A_1^2, B_1^1) $(A_1^1, B_1^1+\tau)$ (A_1^1, B_1^2) $(A_1^1-\tau, B_1^1)$	BY BDO BMO BDO BMO	SY SDO SMO SDO SMO
Case I_5 $\frac{\kappa-k_1}{\kappa-\frac{1}{2}} < \theta_2^I \leq \frac{\kappa-k_1+1}{\kappa+\frac{1}{2}}$	(A_1^1, B_1^1) (A_1^2, B_1^1) $(A_1^1, B_1^1+\tau)$ (A_1^1, B_1^2) $(A_1^1-\tau, B_1^1)$	BY BDO BDO BDO BMO	SY SDO SMO SDO SDO
Case I_6 $\frac{\kappa-k_1+1}{\kappa+\frac{1}{2}} < \theta_2^I$	(A_1^1, B_1^1) (A_1^2, B_1^1) $(A_1^1, B_1^1+\tau)$ (A_1^1, B_1^2) $(A_1^1-\tau, B_1^1)$	BY BDO BDO BDO BDO	SY SDO SDO SDO SDO

Table II.10: Optimal strategies of informed traders at $t = 2$ when the strategy profile at $t = 1$ is (BMO, SMO, BLO, SLO)

Fourth step. Given the optimal response of traders at $t = 2$, we find the optimal action for the traders at $t = 1$ in each of the 6 cases. However, given the nature of this particular equilibrium, we can group cases and analyze them in the following way:

Case $I_1 + I_2 + I_3 : \theta_2^I \leq \dfrac{\kappa - k_1}{\kappa}$

- *Informed traders*

  As $\theta_2^I \leq \dfrac{\kappa - k_1}{\kappa}$, informed traders at $t = 1$ have no incentives to deviate from the prescribed strategy profile whenever

$$
\begin{aligned}
\kappa - k_1 &\geq \frac{1-\lambda}{2}\delta\left(\kappa + k_1 - 1\right) \text{ and} \\
\kappa - k_1 &\geq \theta_1^I\kappa + (1-\theta_1^I)\delta\left(\kappa - k_1 + \lambda\frac{(1-\pi)}{2}I_{SLO,2}^{US,\mathcal{B}_1=\varnothing} - (k_2 - k_1)\left(\lambda\pi I_{BMO,2}^{IH,\mathcal{B}_1=\varnothing} + \frac{1-\lambda}{2}\right)\right).
\end{aligned}
$$

- *Uninformed traders*

  As $\theta_2^I \leq \dfrac{\kappa - k_1}{\kappa} \leq \dfrac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}}$, uninformed traders at $t = 1$ have no incentives to deviate from the prescribed strategy profile whenever

$$
(1-\lambda)\left(k_1 - 1\right) - \lambda\pi\left(\kappa - (k_1 - 1)\right) > 0.
$$

Case $I_4 + I_5 + I_6 : \dfrac{\kappa - k_1}{\kappa} < \theta_2^I$

- *Informed traders*

Consider an informed buyer at $t = 1$. If he chooses a BMO, then he obtains

$$
\mathbb{E}\left(\Pi_{BMO,1}^{IH}\right) = \left(\kappa - k_1\right)\tau.
$$

If instead he deviates towards a BDO, he will obtain

$$
\begin{aligned}
\mathbb{E}\left(\Pi_{BDO,1}^{IH}\right) &= \theta_1^I\kappa\tau + (1-\theta_1^I)\delta\left[\lambda\frac{(1-\pi)}{2}I_{SLO,2}^{US,\mathcal{B}_1=\varnothing} + (\kappa - k_1)\right. \\
&\qquad \left. -(k_2 - k_1)\left(\lambda\pi I_{BMO,2}^{IH,\mathcal{B}_1=\varnothing} + \frac{1-\lambda}{2}\right)\right]\tau.
\end{aligned}
$$

Combining the previous expression and the fact that $\dfrac{\kappa - k_1}{\kappa} < \theta_2^I = \theta_1^I$, it follows that

$$
\mathbb{E}\left(\Pi_{BDO,1}^{IH}\right) > \mathbb{E}\left(\Pi_{BMO,1}^{IH}\right)
$$

is always satisfied and, hence, we conclude that in this case there is no equilibrium in which (BMO, SMO, BLO, SLO) is the strategy profile chosen at $t = 1$.

Fifth step. Based on the above, nobody at $t = 1$ has unilateral incentives to deviate whenever

$$\theta_1^I \leq \frac{\kappa - k_1}{\kappa},$$
$$(1-\lambda)(k_1 - 1) - \lambda\pi(\kappa - (k_1 - 1)) > 0,$$
$$\kappa - k_1 \geq \frac{1-\lambda}{2}\delta(\kappa + k_1 - 1) \text{ and}$$
$$\kappa - k_1 \geq \theta_1^I \kappa + (1-\theta_1^I)\delta\left(\lambda\frac{(1-\pi)}{2} I_{SLO,2}^{US,\mathcal{B}_1=\varnothing} + (\kappa - k_1) - (k_2 - k_1)\left(\lambda\pi I_{BMO,2}^{IH,\mathcal{B}_1=\varnothing} + \frac{1-\lambda}{2}\right)\right).$$

These conditions are equivalent to

$$\theta_1^I \leq \underline{\theta},$$
$$PIN < \psi_{LO-NT}^U,$$
$$\sigma \geq \kappa_{MO-LO}^I \tau, \text{ and}$$
$$\theta_1^I \leq \widehat{\theta}_{MO-DO},$$

where $\underline{\theta}$ is defined in ($II.7$) and

$$\widehat{\theta}_{MO-DO} \equiv \frac{\kappa - k_1 - \delta\left(\lambda\frac{(1-\pi)}{2} I_{SLO,2}^{US,\mathcal{B}_1=\varnothing} + (\kappa - k_1) - (k_2 - k_1)\left(\lambda\pi I_{BMO,2}^{IH,\mathcal{B}_1=\varnothing} + \frac{1-\lambda}{2}\right)\right)}{\kappa - \delta\left(\lambda\frac{(1-\pi)}{2} I_{SLO,2}^{US,\mathcal{B}_1=\varnothing} + (\kappa - k_1) - (k_2 - k_1)\left(\lambda\pi I_{BMO,2}^{IH,\mathcal{B}_1=\varnothing} + \frac{1-\lambda}{2}\right)\right)}.$$

Notice that it can be proved that $\widehat{\theta}_{MO-DO} \leq \underline{\theta}$ and, therefore, we can simplify further to

$$\sigma \geq \kappa_{MO-LO}^I \tau, \ PIN < \psi_{LO-NT}^U \text{ and } \theta_1^I \leq \widehat{\theta}_{MO-DO}. \tag{II.8}$$

Finally, in the following tables we include the moves that are in the equilibrium path taking into account the conditions that must be satisfied if (BMO, SMO, BLO, SLO) is the strategy profile chosen at $t = 1$ and the fact that in this case $\theta_2^I = \theta_1^I$.

Concerning uninformed traders notice that the condition $(1-\lambda)(k_1 - 1) - \lambda\pi(\kappa - (k_1 - 1)) > 0$ implies that $X^{1,D}\kappa < k_1 - 1 < k_2$. Hence, the optimal choice of uninformed traders is

Condition	Optimal Choice of Uninformed Traders at t =2		
	State of the Book	**UB**	**US**
Case $U_1^{\mathcal{E}_1^D}$ $k_1 - 1 \leq \frac{k_2-k_1}{2}$ or $k_1 - 1 > \frac{k_2-k_1}{2}$ and $X^{1,D}\kappa < \frac{k_2-k_1}{2}$	(A_1^2, B_1^1)	NT	SDO
	$(A_1^1, B_1^1+\tau)$	NT	SDO
	(A_1^1, B_1^2)	BDO	NT
	$(A_1^1-\tau, B_1^1)$	BDO	NT
Case $U_2^{\mathcal{E}_1^D}$ $k_1 - 1 > \frac{k_2-k_1}{2}$ and $X^{1,D}\kappa = \frac{k_2-k_1}{2}$	(A_1^2, B_1^1)	NT	NT
	$(A_1^1, B_1^1+\tau)$	NT	SDO
	(A_1^1, B_1^2)	NT	NT
	$(A_1^1-\tau, B_1^1)$	BDO	NT
Case $U_3^{\mathcal{E}_1^D}$ $k_1 - 1 > \frac{k_2-k_1}{2}$ and $\frac{k_2-k_1}{2} < X^{1,D}\kappa < k_1 - 1$	(A_1^2, B_1^1)	BDO	NT
	$(A_1^1, B_1^1+\tau)$	NT	SDO
	(A_1^1, B_1^2)	NT	SDO
	$(A_1^1-\tau, B_1^1)$	BDO	NT

Table II.11: Optimal choice of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is (BMO, SMO, BLO, SLO)

In relation to informed traders:

Condition	Optimal Choice of Informed Traders at t =2		
	State of the Book	**IH**	**IL**
Case I_1 $\theta_1^I \leq \frac{\kappa-k_2}{\kappa-\frac{k_2-k_1}{2}}$	(A_1^2, B_1^1)	BMO	SMO
	$(A_1^1, B_1^1+\tau)$	BMO	SMO
	(A_1^1, B_1^2)	BMO	SMO
	$(A_1^1-\tau, B_1^1)$	BMO	SMO
Case I_2 $\frac{\kappa-k_2}{\kappa-\frac{k_2-k_1}{2}} < \theta_1^I \leq \frac{\kappa-k_1}{\kappa+\frac{k_2-k_1}{2}}$	(A_1^2, B_1^1)	BDO	SMO
	$(A_1^1, B_1^1+\tau)$	BMO	SMO
	(A_1^1, B_1^2)	BMO	SDO
	$(A_1^1-\tau, B_1^1)$	BMO	SMO
Case I_3 $\frac{\kappa-k_1}{\kappa+\frac{k_2-k_1}{2}} < \theta_1^I \leq \frac{\kappa-k_1}{\kappa}$	(A_1^2, B_1^1)	BDO	SDO
	$(A_1^1, B_1^1+\tau)$	BMO	SMO
	(A_1^1, B_1^2)	BDO	SDO
	$(A_1^1-\tau, B_1^1)$	BMO	SMO

Table II.12: Optimal choice of informed traders at $t = 2$ when the strategy profile at $t = 1$ is (BMO, SMO, BLO, SLO)

$\mathcal{E}_2^D : (BMO, SMO, NT, NT)$

For the next five equilibria we provide a sketch of proof. A detailed proof could be provided on request by the authors.

In this case $\Omega_0 = 0$, $\Omega_1 = 1$, $\Omega_2 = 0$, $\Omega_3 = 0$, $\Gamma_0 = 1$, $\Gamma_1 = 0$, $\Gamma_2 = 0$, and $\Gamma_3 = 0$. Moreover, $\theta_2^I = \theta_1^I$ and $\theta_2^U = \theta_1^U$.

Using Bayes' rule we obtain the following beliefs

$$\begin{aligned} X^{2,D} &= \frac{\lambda\pi}{1-\lambda+\lambda\pi},\ Y^{2,D} = y \in [0,1],\ Z^{2,D} = z \in [0,1], \\ p_{BLO,2}^{UB,\mathcal{B}_1=\varnothing} &= p_{SLO,2}^{US,\mathcal{B}_1=\varnothing} = 0, \text{ and } p_{BLO,2}^{IH,\mathcal{B}_1=\varnothing} = p_{SLO,2}^{IL,\mathcal{B}_1=\varnothing} = 0. \end{aligned}$$

We show that nobody at $t = 1$ has unilateral incentives to deviate whenever

$$\begin{gathered} \theta_1^I \leq \frac{\kappa - k_1}{\kappa}, \\ (1-\lambda)(k_1-1) - \lambda\pi(\kappa - (k_1-1)) \leq 0, \\ \kappa - k_1 \geq \frac{1-\lambda}{2}\delta(\kappa + k_1 - 1) \text{ and} \\ \kappa - k_1 \geq \theta_1^I\kappa + (1-\theta_1^I)\delta\left(\kappa - k_1 - (k_2-k_1)\left(\lambda\pi + \frac{1-\lambda}{2}\right)\right). \end{gathered}$$

These conditions are equivalent to:

$$\begin{gathered} \theta_1^I \leq \underline{\theta}, \\ PIN \geq \psi_{LO-NT}^U, \\ \sigma \geq \kappa_{MO-LO}^I\tau, \text{ and} \\ \theta_1^I \leq \overline{\theta}_{MO-DO}, \end{gathered}$$

where $\underline{\theta}$ is defined in ($II.7$) and

$$\overline{\theta}_{MO-DO} \equiv \frac{\kappa - k_1 - \delta\left(\kappa - k_1 - (k_2-k_1)\left(\lambda\pi + \frac{1-\lambda}{2}\right)\right)}{\kappa - \delta\left(\kappa - k_1 - (k_2-k_1)\left(\lambda\pi + \frac{1-\lambda}{2}\right)\right)}. \tag{II.9}$$

Notice also that it can be easily proved that $\overline{\theta}_{MO-DO} \leq \underline{\theta}$ and, therefore, we can simplify further to

$$\sigma \geq \kappa_{MO-LO}^I\tau,\ PIN \geq \psi_{LO-NT}^U, \text{ and } \theta_1^I \leq \overline{\theta}_{MO-DO}. \tag{II.10}$$

Finally, in the following tables we include the decisions that are in the equilibrium path taking into account the conditions that must be satisfied if (BMO, SMO, NT, NT) is the strategy profile chosen at $t = 1$ and that in this case $\theta_2^I = \theta_1^I$.

In relation to uninformed traders, and taking into account that a necessary condition for this equilibrium tells us that $k_1 - 1 \leq X^{2,D}\kappa$, the following cases can be distinguished:

Condition	Optimal Choice of Uninformed Traders at t =2		
	State of the Book	**UB**	**US**
Case $U_1^{\mathcal{E}_2^D}$ $k_1 - 1 \leq X^{2,D}\kappa < \frac{k_2-k_1}{2}$	(A_1^1, B_1^1) (A_1^2, B_1^1) (A_1^1, B_1^2)	NT NT BDO	NT SDO NT
Case $U_2^{\mathcal{E}_2^D}$ $k_1 - 1 < X^{2,D}\kappa = \frac{k_2-k_1}{2}$	(A_1^1, B_1^1) (A_1^2, B_1^1) (A_1^1, B_1^2)	NT NT NT	NT NT NT
Case $U_3^{\mathcal{E}_2^D}$ $\max\left\{k_1 - 1, \frac{k_2-k_1}{2}\right\} < X^{2,D}\kappa \leq k_2$ or $k_2 < X^{2,D}\kappa$ and $\theta_2^U > \theta_{X^{2,D}}$	(A_1^1, B_1^1) (A_1^2, B_1^1) (A_1^1, B_1^2)	NT BDO NT	NT NT SDO
Case $U_4^{\mathcal{E}_2^D}$ $k_2 < X^{2,D}\kappa$ and $\theta_2^U \leq \theta_{X^{2,D}}$	(A_1^1, B_1^1) (A_1^2, B_1^1) (A_1^1, B_1^2)	NT BMO NT	NT NT SMO

Table II.13: Optimal choice of uniformed traders at $t = 2$ when the strategy profile at $t = 1$ is (BMO, SMO, NT, NT)

In relation to informed traders:

Condition	Optimal Choice of Informed Traders at t =2		
	State of the Book	**IH**	**IL**
Case I_1 $\theta_1^I \leq \frac{\kappa-k_2}{\kappa-\frac{k_2-k_1}{2}}$	(A_1^1, B_1^1) (A_1^2, B_1^1) (A_1^1, B_1^2)	BMO BMO BMO	SMO SMO SMO
Case I_2 $\frac{\kappa-k_2}{\kappa-\frac{k_2-k_1}{2}} < \theta_1^I \leq \frac{\kappa-k_1}{\kappa+\frac{k_2-k_1}{2}}$	(A_1^1, B_1^1) (A_1^2, B_1^1) (A_1^1, B_1^2)	BMO BDO BMO	SMO SMO SDO
Case I_3 $\frac{\kappa-k_1}{\kappa+\frac{k_2-k_1}{2}} < \theta_1^I \leq \frac{\kappa-k_1}{\kappa}$	(A_1^1, B_1^1) (A_1^2, B_1^1) (A_1^1, B_1^2)	BMO BDO BDO	SMO SDO SDO

Table II.14: Optimal choice of informed traders at $t = 2$ when the strategy profile at $t = 1$ is (BMO, SMO, NT, NT)

$\mathcal{E}_3^D : (BLO, SLO, BLO, BLO)$

In this case $\Omega_0 = 0$, $\Omega_1 = 0$, $\Omega_2 = 1$, $\Omega_3 = 0$, $\Gamma_0 = 0$, $\Gamma_1 = 0$, $\Gamma_2 = 1$, and $\Gamma_3 = 0$. Moreover, $\theta_1^I = \theta_2^I$ and $\theta_1^U = \theta_2^U$.

Using Bayes' rule we obtain the following beliefs

$$\begin{aligned} X^{3,D} &= 0,\ Y^{3,D} = \pi \text{ and } Z^{3,D} = z \in [0,1], \\ p_{BLO,2}^{UB,\mathcal{B}_1=\varnothing} &= p_{SLO,2}^{US,\mathcal{B}_1=\varnothing} \in [0,1], \text{ and } p_{BLO,2}^{IH,\mathcal{B}_1=\varnothing} = p_{SLO,2}^{IL,\mathcal{B}_1=\varnothing} \in [0,1]. \end{aligned}$$

The conditions under which nobody is willing to deviate at $t = 1$ are:

$$\begin{aligned} &\sigma < \kappa_{MO-LO}^I \tau,\ PIN < \psi_{LO-NT}^U, \text{ and } \theta_1^I \le \min\{\underline{\theta}, \widehat{\theta}_{LO-DO}\}, \\ \text{or} \quad & PIN < \psi_{LO-NT}^U \text{ and } \underline{\theta} < \theta_1^I \le \min\{\overline{\theta}, \underline{\theta}_{LO-DO}\}, \\ \text{or} \quad & k_1 > 1 \text{ and } \overline{\theta} < \theta_1^I \le \underline{\theta}_{LO-DO}. \end{aligned} \tag{II.11}$$

Finally, in the following tables we include the moves that are in the equilibrium path taking into account the conditions that must be satisfied if (BLO, SLO, BLO, SLO) is the strategy profile chosen at $t = 1$ and the fact that in this case $\theta_2^I = \theta_1^I$.

Concerning uninformed traders, it follows that

Condition	**Optimal Choice of U**
	State of the Book
Case $U_1^{\mathcal{E}_3^D}$ $Y^{3,D}\kappa \le \frac{1}{2}$	(A_1^2, B_1^1) $(A_1^1, B_1^1 + \tau)$ (A_1^1, B_1^2) $(A_1^1 - \tau, B_1^1)$
Case $U_2^{\mathcal{E}_3^D}$ $\frac{1}{2} < Y^{3,D}\kappa \le k_1$ or $Y^{3,D}\kappa > k_1$and $\theta_2^U > \theta_{Y^{3,D}}$	(A_1^2, B_1^1) $(A_1^1, B_1^1 + \tau)$ (A_1^1, B_1^2) $(A_1^1 - \tau, B_1^1)$
Case $U_3^{\mathcal{E}_3^D}$ $Y^{3,D}k_3 > k_1$ and $\theta_2^U \le \theta_{Y^{3,D}}$	(A_1^2, B_1^1) $(A_1^1, B_1^1 + \tau)$ (A_1^1, B_1^2) $(A_1^1 - \tau, B_1^1)$

In relation to informed traders:

Condition	Optimal Choice of Informed Traders at t =2		
	State of the Book	**IH**	**IL**
Case I_1 $\theta_1^I \leq \frac{\kappa-k_2}{\kappa-\frac{k_2-k_1}{2}}$	(A_1^2, B_1^1)	BMO	SMO
	$(A_1^1, B_1^1+\tau)$	BMO	SMO
	(A_1^1, B_1^2)	BMO	SMO
	$(A_1^1-\tau, B_1^1)$	BMO	SMO
Case I_2 $\frac{\kappa-k_2}{\kappa-\frac{k_2-k_1}{2}} < \theta_1^I \leq \frac{\kappa-k_1}{\kappa+\frac{k_2-k_1}{2}}$	(A_1^2, B_1^1)	BDO	SMO
	$(A_1^1, B_1^1+\tau)$	BMO	SMO
	(A_1^1, B_1^2)	BMO	SDO
	$(A_1^1-\tau, B_1^1)$	BMO	SMO
Case I_3 $\frac{\kappa-k_1}{\kappa+\frac{k_2-k_1}{2}} < \theta_1^I \leq \frac{\kappa-k_1}{\kappa}$	(A_1^2, B_1^1)	BDO	SDO
	$(A_1^1, B_1^1+\tau)$	BMO	SMO
	(A_1^1, B_1^2)	BDO	SDO
	$(A_1^1-\tau, B_1^1)$	BMO	SMO
Case I_4 $\frac{\kappa-k_1}{\kappa} < \theta_1^I \leq \frac{\kappa-k_1}{\kappa-\frac{1}{2}}$	(A_1^2, B_1^1)	BDO	SDO
	$(A_1^1, B_1^1+\tau)$	BMO	SMO
	(A_1^1, B_1^2)	BDO	SDO
	$(A_1^1-\tau, B_1^1)$	BMO	SMO
Case I_5 $\frac{\kappa-k_1}{\kappa-\frac{1}{2}} < \theta_1^I \leq \frac{\kappa-k_1+1}{\kappa+\frac{1}{2}}$	(A_1^2, B_1^1)	BDO	SDO
	$(A_1^1, B_1^1+\tau)$	BDO	SMO
	(A_1^1, B_1^2)	BDO	SDO
	$(A_1^1-\tau, B_1^1)$	BMO	SDO
Case I_6 $\frac{\kappa-k_1+1}{\kappa+\frac{1}{2}} < \theta_1^I$	(A_1^2, B_1^1)	BDO	SDO
	$(A_1^1, B_1^1+\tau)$	BDO	SDO
	(A_1^1, B_1^2)	BDO	SDO
	$(A_1^1-\tau, B_1^1)$	BDO	SDO

Table II.15: Optimal choice of informed traders when the strategy profile at $t = 1$ is (BLO, SLO, BLO, SLO).

$\mathcal{E}_4^D : (BLO, SLO, NT, NT)$

In this case $\Omega_0 = 0$, $\Omega_1 = 0$, $\Omega_2 = 1$, $\Omega_3 = 0$, $\Gamma_0 = 1$, $\Gamma_1 = 0$, $\Gamma_2 = 0$, and $\Gamma_3 = 0$. Moreover, $\theta_1^I = \theta_2^I$ and $\theta_1^U = \theta_2^U$.

Using Bayes' rule

$$\begin{aligned} X^{4,D} &= 0,\ Y^{4,D} = 1,\ Z^{4,D} = z \in [0,1]\,, \\ p_{BLO,2}^{UB,\mathcal{B}_1=\varnothing} &= p_{SLO,2}^{US,\mathcal{B}_1=\varnothing} = 0, \text{ and } p_{BLO,2}^{IH,\mathcal{B}_1=\varnothing} = p_{SLO,2}^{IL,\mathcal{B}_1=\varnothing} = 0. \end{aligned}$$

The conditions under which nobody is willing to deviate at $t = 1$ are:

$$\begin{aligned} &\sigma < \kappa_{MO-LO}^I \tau,\ PIN \geq \psi_{LO-NT}^U, \text{ and } \theta_1^I \leq \min\{\underline{\theta}, \overline{\theta}_{LO-DO}\}, \\ \text{or} \quad &PIN \geq \psi_{LO-NT}^U \text{ and } \underline{\theta} < \theta_1^I \leq \min\{\overline{\theta}, \widetilde{\theta}_{LO-DO}\}. \end{aligned} \tag{II.12}$$

Finally, in the following tables we include the moves that are in the equilibrium path taking into account the conditions that must be satisfied if (BLO, SLO, NT, NT) is the strategy profile chosen at $t = 1$ and the fact that in this case $\theta_2^I = \theta_1^I$ and $\theta_2^U = \theta_1^U$.

Concerning the uninformed traders, we have

Condition	**Optimal Choice of Uninformed Traders at t =2**		
	State of the Book	**UB**	**US**
Case $U_1^{\mathcal{E}_4^D}$ $\theta_2^U > \theta_{Y^{4,D}}$	(A_1^1, B_1^1)	NT	NT
	(A_1^2, B_1^1)	NT	SDO
	$(A_1^1, B_1^1 + \tau)$	BMO	NT
	(A_1^1, B_1^2)	BDO	NT
	$(A_1^1 - \tau, B_1^1)$	NT	SMO
Case $U_2^{\mathcal{E}_4^D}$ $\theta_2^U \leq \theta_{Y^{4,D}}$	(A_1^1, B_1^1)	NT	NT
	(A_1^2, B_1^1)	NT	SDO
	$(A_1^1, B_1^1 + \tau)$	BDO	NT
	(A_1^1, B_1^2)	BDO	NT
	$(A_1^1 - \tau, B_1^1)$	NT	SDO

Table II.16: Optimal choice of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is (BLO, SLO, NT, NT).

For informed traders, we have:

Condition	Optimal Choice of Informed Traders at t =2		
	State of the Book	**IH**	**IL**
Case I_1 $\theta_1^I \le \frac{\kappa-k_2}{\kappa-\frac{k_2-k_1}{2}}$	(A_1^1, B_1^1)	BMO	SMO
	(A_1^2, B_1^1)	BMO	SMO
	$(A_1^1, B_1^1+\tau)$	BMO	SMO
	(A_1^1, B_1^2)	BMO	SMO
	$(A_1^1-\tau, B_1^1)$	BMO	SMO
Case I_2 $\frac{\kappa-k_2}{\kappa-\frac{k_2-k_1}{2}} < \theta_1^I \le \frac{\kappa-k_1}{\kappa+\frac{k_2-k_1}{2}}$	(A_1^1, B_1^1)	BMO	SMO
	(A_1^2, B_1^1)	BDO	SMO
	$(A_1^1, B_1^1+\tau)$	BMO	SMO
	(A_1^1, B_1^2)	BMO	SDO
	$(A_1^1-\tau, B_1^1)$	BMO	SMO
Case I_3 $\frac{\kappa-k_1}{\kappa+\frac{k_2-k_1}{2}} < \theta_1^I \le \frac{\kappa-k_1}{\kappa}$	(A_1^1, B_1^1)	BMO	SMO
	(A_1^2, B_1^1)	BDO	SDO
	$(A_1^1, B_1^1+\tau)$	BMO	SMO
	(A_1^1, B_1^2)	BDO	SDO
	$(A_1^1-\tau, B_1^1)$	BMO	SMO
Case I_4 $\frac{\kappa-k_1}{\kappa} < \theta_1^I \le \frac{\kappa-k_1}{\kappa-\frac{1}{2}}$	(A_1^1, B_1^1)	BDO	SDO
	(A_1^2, B_1^1)	BDO	SDO
	$(A_1^1, B_1^1+\tau)$	BMO	SMO
	(A_1^1, B_1^2)	BDO	SDO
	$(A_1^1-\tau, B_1^1)$	BMO	SMO
Case I_5 $\frac{\kappa-k_1}{\kappa-\frac{1}{2}} < \theta_1^I \le \frac{\kappa-k_1+1}{\kappa+\frac{1}{2}}$	(A_1^1, B_1^1)	BDO	SDO
	(A_1^2, B_1^1)	BDO	SDO
	$(A_1^1, B_1^1+\tau)$	BDO	SMO
	(A_1^1, B_1^2)	BDO	SDO
	$(A_1^1-\tau, B_1^1)$	BMO	SDO
Case I_6 $\frac{\kappa-k_1+1}{\kappa+\frac{1}{2}} < \theta_1^I$	(A_1^1, B_1^1)	BDO	SDO
	(A_1^2, B_1^1)	BDO	SDO
	$(A_1^1, B_1^1+\tau)$	BDO	SDO
	(A_1^1, B_1^2)	BDO	SDO
	$(A_1^1-\tau, B_1^1)$	BDO	SDO

Table II.17: Optimal choice of informed traders at $t = 2$ when the strategy profile at $t = 1$ is (BLO, SLO, NT, NT).

$\mathcal{E}_5^D$: (BDO, SDO, BLO, SLO)
In this case $\Omega_0 = 0$, $\Omega_1 = 0$, $\Omega_2 = 0$, $\Omega_3 = 1$, $\Gamma_0 = 0$, $\Gamma_1 = 0$, $\Gamma_2 = 1$, and $\Gamma_3 = 0$. Moreover, $\theta_2^I \le \theta_1^I$ and $\theta_2^U \le \theta_1^U$.
Using Bayes' rule we obtain $X^{5,D} = 0$, $Y^{5,D} = 0$ and $Z^{5,D} = 1$.

We show that nobody at $t = 1$ has unilateral incentives to deviate from (BDO, SDO, BLO, SLO) whenever

$$\begin{aligned} & PIN < \psi^U_{LO-NT},\ \theta^I_1 > \max\{\overline{\theta}_{MO-DO}, \overline{\theta}_{LO-DO}\},\text{ and } \theta^I_2 \leq \min\left\{\underline{\theta}, \theta^I_1\right\}, \\ \text{or} \quad & PIN < \psi^U_{LO-NT},\ \widetilde{\theta}_{LO-DO} < \theta^I_1,\text{ and } \underline{\theta} < \theta^I_2 \leq \min\left\{\overline{\theta}, \theta^I_1\right\}, \\ \text{or} \quad & k_1 > 1,\ \widetilde{\theta}_{LO-DO} < \theta^I_1,\text{ and } \overline{\theta} < \theta^I_2 \leq \theta^I_1. \end{aligned} \tag{II.13}$$

Finally, we consider only the moves that are in the equilibrium path taking into account the conditions that must be satisfied if (BDO, SDO, BLO, SLO) is the strategy profile chosen at $t = 1$. For uninformed traders,

Optimal Choice of Uninformed Traders at $t = 2$		
State of the Book	**UB**	**US**
(A^1_1, B^1_1)	NT	NT
(A^2_1, B^1_1)	NT	SDO
$(A^1_1, B^1_1 + \tau)$	NT	SDO
(A^1_1, B^2_1)	BDO	NT
$(A^1_1 - \tau, B^1_1)$	BDO	NT

Table II.18: Optimal choice of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is (BDO, SDO, BLO, SLO).

For informed traders,

Condition	Optimal Choice of Informed Traders at t =2		
	State of the Book	**IH**	**IL**
Case I_1 $\theta_2^I \leq \frac{\kappa-k_2}{\kappa-\frac{k_2-k_1}{2}}$	(A_1^1, B_1^1)	BMO	SMO
	(A_1^2, B_1^1)	BMO	SMO
	$(A_1^1, B_1^1+\tau)$	BMO	SMO
	(A_1^1, B_1^2)	BMO	SMO
	$(A_1^1-\tau, B_1^1)$	BMO	SMO
Case I_2 $\frac{\kappa-k_2}{\kappa-\frac{k_2-k_1}{2}} < \theta_2^I \leq \frac{\kappa-k_1}{\kappa+\frac{k_2-k_1}{2}}$	(A_1^1, B_1^1)	BMO	SMO
	(A_1^2, B_1^1)	BDO	SMO
	$(A_1^1, B_1^1+\tau)$	BMO	SMO
	(A_1^1, B_1^2)	BMO	SDO
	$(A_1^1-\tau, B_1^1)$	BMO	SMO
Case I_3 $\frac{\kappa-k_1}{\kappa+\frac{k_2-k_1}{2}} < \theta_2^I \leq \frac{\kappa-k_1}{\kappa}$	(A_1^1, B_1^1)	BMO	SMO
	(A_1^2, B_1^1)	BDO	SDO
	$(A_1^1, B_1^1+\tau)$	BMO	SMO
	(A_1^1, B_1^2)	BDO	SDO
	$(A_1^1-\tau, B_1^1)$	BMO	SMO
Case I_4 $\frac{\kappa-k_1}{\kappa} < \theta_2^I \leq \frac{\kappa-k_1}{\kappa-\frac{1}{2}}$	(A_1^1, B_1^1)	BDO	SDO
	(A_1^2, B_1^1)	BDO	SDO
	$(A_1^1, B_1^1+\tau)$	BMO	SMO
	(A_1^1, B_1^2)	BDO	SDO
	$(A_1^1-\tau, B_1^1)$	BMO	SMO
Case I_5 $\frac{\kappa-k_1}{\kappa-\frac{1}{2}} < \theta_2^I \leq \frac{\kappa-k_1+1}{\kappa+\frac{1}{2}}$	(A_1^1, B_1^1)	BDO	SDO
	(A_1^2, B_1^1)	BDO	SDO
	$(A_1^1, B_1^1+\tau)$	BDO	SMO
	(A_1^1, B_1^2)	BDO	SDO
	$(A_1^1-\tau, B_1^1)$	BMO	SDO
Case I_6 $\frac{\kappa-k_1+1}{\kappa+\frac{1}{2}} < \theta_2^I$	(A_1^1, B_1^1)	BDO	SDO
	(A_1^2, B_1^1)	BDO	SDO
	$(A_1^1, B_1^1+\tau)$	BDO	SDO
	(A_1^1, B_1^2)	BDO	SDO
	$(A_1^1-\tau, B_1^1)$	BDO	SDO

Table II.19: Optimal choice of informed traders at $t = 2$ when the strategy profile at $t = 1$ is (BDO, SDO, BLO, SLO).

$\mathcal{E}_6^D : (BDO, SDO, NT, NT)$
In this case $\Omega_0 = 0$, $\Omega_1 = 0$, $\Omega_2 = 0$, $\Omega_3 = 1$, $\Gamma_0 = 1$, $\Gamma_1 = 0$, $\Gamma_2 = 0$, and $\Gamma_3 = 0$. Moreover, $\theta_2^I \leq \theta_1^I$ and $\theta_2^U \leq \theta_1^U$.

Using Bayes' rule, we obtain

$$
\begin{aligned}
X^{6,D} &= 0,\ Y^{6,D} = y \in [0,1] \text{ and } Z^{6,D} = 1, \\
p_{BLO,2}^{UB,\mathcal{B}_1=\varnothing} &= p_{SLO,2}^{US,\mathcal{B}_1=\varnothing} = (1-\theta_1^I)\frac{\pi}{2}, \text{ and } p_{BLO,2}^{IH,\mathcal{B}_1=\varnothing} = p_{SLO,2}^{IL,\mathcal{B}_1=\varnothing} = 0
\end{aligned}
$$

Nobody at $t = 1$ has unilateral incentives to deviate from (BDO, SDO, NT, NT) whenever

$$
\begin{aligned}
& PIN \geq \psi_{LO-NT}^U,\ \theta_1^I > \max\{\overline{\theta}_{MO-DO}, \overline{\theta}_{LO-DO}\}, \text{ and } \theta_2^I \leq \underline{\theta}, \\
\text{or} \quad & PIN \geq \psi_{LO-NT}^U,\ \widetilde{\theta}_{LO-DO} < \theta_1^I, \text{ and } \underline{\theta} < \theta_2^I \leq \overline{\theta}, \\
\text{or} \quad & \widetilde{\theta}_{LO-DO} < \theta_1^I,\ \overline{\theta} < \theta_2^I \leq \theta_1^I \text{ and } k_1 = 1.
\end{aligned}
\tag{II.14}
$$

Finally, in the following tables we include the moves that are in the equilibrium path taking into account the conditions that must be satisfied if (BDO, SDO, NT, NT) is the strategy profile chosen at $t = 1$.

For uninformed traders,

Optimal Choice of Uninformed Traders at $t = 2$		
State of the Book	**UB**	**US**
(A_1^1, B_1^1)	NT	NT
(A_1^2, B_1^1)	NT	SDO
(A_1^1, B_1^2)	BDO	NT

Table II.20: Optimal choice of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is (BDO, SDO, NT, NT).

For informed traders,

Condition	Optimal Choice of Informed Traders at t =2		
	State of the Book	IH	IL
Case I_1 $\theta_2^I \le \frac{\kappa-k_2}{\kappa-\frac{k_2-k_1}{2}}$	(A_1^1, B_1^1) (A_1^2, B_1^1) (A_1^1, B_1^2)	BMO BMO BMO	SMO SMO SMO
Case I_2 $\frac{\kappa-k_2}{\kappa-\frac{k_2-k_1}{2}} < \theta_2^I \le \frac{\kappa-k_1}{\kappa+\frac{k_2-k_1}{2}}$	(A_1^1, B_1^1) (A_1^2, B_1^1) (A_1^1, B_1^2)	BMO BDO BMO	SMO SMO SDO
Case I_3 $\frac{\kappa-k_1}{\kappa+\frac{k_2-k_1}{2}} < \theta_2^I \le \frac{\kappa-k_1}{\kappa}$	(A_1^1, B_1^1) (A_1^2, B_1^1) (A_1^1, B_1^2)	BMO BDO BDO	SMO SDO SDO
Case I_4 $\frac{\kappa-k_1}{\kappa} < \theta_2^I \le \frac{\kappa-k_1}{\kappa-\frac{1}{2}}$	(A_1^1, B_1^1) (A_1^2, B_1^1) (A_1^1, B_1^2)	BDO BDO BDO	SDO SDO SDO
Case I_5 $\frac{\kappa-k_1}{\kappa-\frac{1}{2}} < \theta_2^I \le \frac{\kappa-k_1+1}{\kappa+\frac{1}{2}}$	(A_1^1, B_1^1) (A_1^2, B_1^1) (A_1^1, B_1^2)	BDO BDO BDO	SDO SDO SDO
Case I_6 $\frac{\kappa-k_1+1}{\kappa+\frac{1}{2}} < \theta_2^I$	(A_1^1, B_1^1) (A_1^2, B_1^1) (A_1^1, B_1^2)	BDO BDO BDO	SDO SDO SDO

Table II.21: Optimal choice of informed traders at $t = 2$ when the strategy profile at $t = 1$ is (BDO, SDO, NT, NT).

Finally, note that substituting $k_1 = 1$ into the expressions of κ^I_{MO-LO} and ψ^U_{LO-NT}, we have that

$$\kappa^I_{MO-LO} = \frac{1}{1-\frac{1}{2}\delta(1-\lambda)}, \text{ and}$$
$$\psi^U_{LO-NT} = 0.$$

Moreover, since $\kappa^I_{MO-LO} < 2$, it follows that $\kappa^I_{MO-LO}\tau < \sigma$ and $PIN \ge \psi^U_{LO-NT}$. Therefore, using (II.8)-(II.14), we have that when $k_1 = 1$, the conditions related to $\mathcal{E}_1^D$, $\mathcal{E}_3^D$ and $\mathcal{E}_5^D$ do not hold. Moreover, when $k_1 = 1$, $\delta\frac{1-\lambda}{2}\kappa < \kappa - 1$, which implies that an informed trader at $t = 1$ prefers a MO to a LO. Hence, $\mathcal{E}_4^D$ is not feasible when $k_1 = 1$. Therefore, in this case we have that (BMO, SMO, NT, NT) is the optimal strategy profile at $t = 1$ if $\theta_1^I \le \overline{\theta}_{MO-DO}$; and (BDO, SDO, NT, NT) is the optimal strategy profile at $t = 1$ if

$$\theta_1^I > \max\{\overline{\theta}_{MO-DO}, \overline{\theta}_{LO-DO}\} \text{ and } \theta_2^I \le \min\left\{\underline{\theta}, \theta_1^I\right\},$$
$$\text{or} \quad \widetilde{\theta}_{LO-DO} < \theta_1^I \text{ and } \underline{\theta} < \theta_2^I \le \theta_1^I.$$

■

Proof of Proposition 2.

Case A We consider the same four possible cases depending on the initial conditions in the single-venue market.

Case A.1: $\sigma < \kappa^I_{MO-LO}\tau$ and $PIN < \psi^U_{LO-NT}$

In this case, we start with a market in which the equilibrium is $\mathcal{E}_3^{ND}$, where conditions ($II.4$) are satisfied. In this case, when we add the DP out of the 6 equilibria, there are only two possible equilibria that satisfy these conditions: $\mathcal{E}_3^D$ and $\mathcal{E}_5^D$. From Lemma C.2 we can see that $\mathcal{E}_3^D$ is an equilibrium if conditions (II.11) are satisfied. This can be rewritten as

$$\begin{aligned} &\theta_1^I \le \min\{\underline{\theta}, \widehat{\theta}_{LO-DO}\} \\ \text{or} \quad &\underline{\theta} < \theta_1^I \le \underline{\theta}_{LO-DO}. \end{aligned}$$

Using the relationship between cutoffs (C.4) (in the paper), we know that $\underline{\theta}_{LO-DO} \le \widehat{\theta}_{LO-DO}$. Then, we consider the following cases: I) $\underline{\theta} < \underline{\theta}_{LO-DO}$, II) $\underline{\theta}_{LO-DO} \le \underline{\theta} < \widehat{\theta}_{LO-DO}$, and III) $\widehat{\theta}_{LO-DO} \le \underline{\theta}$.

Case I: $\underline{\theta} < \underline{\theta}_{LO-DO}$. As $\underline{\theta}_{LO-DO} \le \widehat{\theta}_{LO-DO}$, it follows that $\underline{\theta} < \widehat{\theta}_{LO-DO}$. Hence, the conditions that guarantee that $\mathcal{E}_3^D$ is an equilibrium can be rewritten as

$$\begin{aligned} &\theta_1^I \le \underline{\theta} \\ \text{or} \quad &\underline{\theta} < \theta_1^I \le \underline{\theta}_{LO-DO}, \end{aligned}$$

which can be further simplified as

$$\theta_1^I \le \underline{\theta}_{LO-DO}.$$

Case II: $\underline{\theta}_{LO-DO} \le \underline{\theta} < \widehat{\theta}_{LO-DO}$. In this case, the conditions that guarantee that $\mathcal{E}_3^D$ is an equilibrium can be rewritten as

$$\theta_1^I \le \underline{\theta}_{LO-DO}.$$

Case III: $\widehat{\theta}_{LO-DO} \le \underline{\theta}$. In this case, the conditions that guarantee that $\mathcal{E}_3^D$ is an equilibrium can be rewritten as

$$\theta_1^I \le \widehat{\theta}_{LO-DO}.$$

Consequently, we conclude that $\mathcal{E}_3^D$ is an equilibrium whenever

$$\begin{aligned} &\theta_1^I \le \underline{\theta}_{LO-DO} && \text{if } \underline{\theta} < \underline{\theta}_{LO-DO}, \\ \text{or} \quad &\theta_1^I \le \underline{\theta} && \text{if } \underline{\theta}_{LO-DO} \le \underline{\theta} < \widehat{\theta}_{LO-DO}, \\ \text{or} \quad &\theta_1^I \le \widehat{\theta}_{LO-DO} && \text{if } \widehat{\theta}_{LO-DO} \le \underline{\theta}. \end{aligned}$$

On the other hand, $\mathcal{E}_5^D$ is an equilibrium if conditions ($II.13$) are satisfied, and in this case they can be rewritten as

$$\begin{aligned} &\theta_1^I > \max\{\overline{\theta}_{MO-DO}, \overline{\theta}_{LO-DO}\} \text{ and } \theta_2^I \leq \min\{\underline{\theta}, \theta_1^I\}, \\ \text{or} \quad &\widetilde{\theta}_{LO-DO} < \theta_1^I \text{ and } \underline{\theta} < \theta_2^I \leq \theta_1^I. \end{aligned}$$

Notice that when $\sigma < \kappa^I_{MO-LO}\tau$, the informed traders prefer LO to MO and, therefore, $\overline{\theta}_{MO-DO} < \overline{\theta}_{LO-DO}$. Hence, $\mathcal{E}_5^D$ the equilibrium if

$$\begin{aligned} &\theta_1^I > \overline{\theta}_{LO-DO} \text{ and } \theta_2^I \leq \min\{\underline{\theta}, \theta_1^I\}, \\ \text{or} \quad &\widetilde{\theta}_{LO-DO} < \theta_1^I \text{ and } \underline{\theta} < \theta_2^I \leq \theta_1^I. \end{aligned}$$

As a result, when $\sigma < \kappa^I_{MO-LO}\tau$ and $PIN < \psi^U_{LO-NT}$, the optimal strategy profiles at $t = 1$ are

$$\begin{cases} (BLO, SLO, BLO, SLO), & \text{if} \quad \theta_1^I \leq \theta^1_{LO-LO}, \\ (BDO, SDO, BLO, BLO) & \text{if} \quad \theta_1^I > \theta^1_{DO-LO}, \end{cases}$$

where

$$\begin{aligned} \theta^1_{LO-LO} &= \begin{cases} \underline{\theta}_{LO-DO} & \text{if } \underline{\theta} < \underline{\theta}_{LO-DO}, \\ \underline{\theta} & \text{if } \underline{\theta}_{LO-DO} \leq \underline{\theta} < \widehat{\theta}_{LO-DO}, \\ \widehat{\theta}_{LO-DO} & \text{otherwise} \end{cases} \quad \text{and} \\ \theta^1_{DO-LO} &= \begin{cases} \overline{\theta}_{LO-DO} & \text{if} \quad \theta_2^I \leq \min\{\underline{\theta}, \theta_1^I\} \\ \widetilde{\theta}_{LO-DO} & \quad \underline{\theta} < \theta_2^I \leq \theta_1^I. \end{cases} \end{aligned} \tag{II.15}$$

Case A.2: $\sigma < \kappa^I_{MO-LO}\tau$ and $PIN \geq \psi^U_{LO-NT}$

In this case, when we start with a market in which the equilibrium is $\mathcal{E}_4^{ND}$, where conditions (**??**) and (**??**) are satisfied. In this case, when we add the DP out of the 6 equilibria there are only three possible equilibria that satisfy these conditions: $\mathcal{E}_4^D$, $\mathcal{E}_5^D$, and $\mathcal{E}_6^D$.[2]

In addition, we can see that $\mathcal{E}_4^D$ is an equilibrium if conditions ($II.12$) are satisfied, and in this case they can be rewritten as

$$\begin{aligned} &\theta_1^I \leq \min\{\underline{\theta}, \overline{\theta}_{LO-DO}\}, \\ \text{or} \quad &\underline{\theta} < \theta_1^I \leq \min\{\overline{\theta}, \widetilde{\theta}_{LO-DO}\}. \end{aligned}$$

Consider the following cases: I) $\underline{\theta} < \overline{\theta}_{LO-DO}$ and II) $\overline{\theta}_{LO-DO} \leq \underline{\theta}$.

Case I: $\underline{\theta} < \overline{\theta}_{LO-DO}$. In this case, the conditions that guarantee that $\mathcal{E}_4^D$ is an equilibrium can be rewritten as

$$\begin{aligned} &\theta_1^I \leq \underline{\theta}, \\ \text{or} \quad &\underline{\theta} < \theta_1^I \leq \min\{\overline{\theta}, \widetilde{\theta}_{LO-DO}\}, \end{aligned}$$

[2] It is straightforward that when the conditions for the equilibrium $\mathcal{E}_4^{ND}$ are satisfied, the equilibria $\mathcal{E}_1^D$ and $\mathcal{E}_2^D$ are not feasible. It can be proved that in this case the equilibrium $\mathcal{E}_3^D$ is also not feasible.

i.e.,

$$\theta_1^I \leq \min\{\overline{\theta}, \widetilde{\theta}_{LO-DO}\}.$$

Case II: $\overline{\theta}_{LO-DO} \leq \underline{\theta}$. In this case, $\widetilde{\theta}_{LO-DO} < \underline{\theta}$. Thus, the conditions that guarantee that $\mathcal{E}_4^D$ is an equilibrium can be rewritten as

$$\theta_1^I \leq \overline{\theta}_{LO-DO}.$$

On the other hand, $\mathcal{E}_5^D$ is an equilibrium if conditions ($II.13$) are satisfied, and in this case they can be rewritten as

$$\widetilde{\theta}_{LO-DO} < \theta_1^I \text{ and } \overline{\theta} < \theta_2^I \leq \theta_1^I.$$

Finally, $\mathcal{E}_6^D$ is an equilibrium if

$$\begin{array}{ll} & \theta_1^I > \max\{\overline{\theta}_{MO-DO}, \overline{\theta}_{LO-DO}\}, \text{ and } \theta_2^I \leq \min\left\{\underline{\theta}, \theta_1^I\right\}, \\ \text{or} & \widetilde{\theta}_{LO-DO} < \theta_1^I, \text{ and } \underline{\theta} < \theta_2^I \leq \min\left\{\overline{\theta}, \theta_1^I\right\}, \end{array}$$

Given that $\sigma < \kappa_{MO-LO}^I \tau$, we know that the informed prefer LO to MO. Hence, $\overline{\theta}_{LO-DO} > \overline{\theta}_{MO-DO}$, which implies $\max\{\overline{\theta}_{MO-DO}, \overline{\theta}_{LO-DO}\} = \overline{\theta}_{LO-DO}$. Thus, the previous conditions can be rewritten as

$$\begin{array}{ll} & \theta_1^I > \overline{\theta}_{LO-DO}, \text{ and } \theta_2^I \leq \min\left\{\underline{\theta}, \theta_1^I\right\}, \\ \text{or} & \widetilde{\theta}_{LO-DO} < \theta_1^I, \text{ and } \underline{\theta} < \theta_2^I \leq \min\left\{\overline{\theta}, \theta_1^I\right\}, \end{array}$$

As a result, the optimal strategy profiles of a trader at $t = 1$ are

$$\begin{cases} (BLO, SLO, NT, NT) & \text{if} \quad \theta_1^I \leq \theta_{LO-NT}^{22} \\ (BDO, SDO, NT, NT) & \text{if} \quad \theta_{DO-NT}^{22} < \theta_1^I \leq \theta_{DO-LO}^{22} \\ (BDO, SDO, BLO, BLO) & \text{if} \quad \theta_1^I > \theta_{DO-LO}^{22}, \end{cases}$$

where

$$\theta_{LO-NT}^{22} = \begin{cases} \min\{\overline{\theta}, \widetilde{\theta}_{LO-DO}\} & \text{if} \quad \underline{\theta} < \overline{\theta}_{LO-DO} \\ \overline{\theta}_{LO-DO} & \text{otherwise,} \end{cases}$$

$$\begin{aligned} \theta_{DO-NT}^{22} &= \begin{cases} \overline{\theta}_{LO-DO} & \text{if} \quad \theta_2^I \leq \min\left\{\underline{\theta}, \theta_1^I\right\} \\ \widetilde{\theta}_{LO-DO} & \text{if} \quad \underline{\theta} < \theta_2^I \leq \min\left\{\overline{\theta}, \theta_1^I\right\} \\ 1 & \text{if} \quad \min\left\{\overline{\theta}, \theta_1^I\right\} < \theta_2^I, \end{cases} \quad \text{and} \\ \theta_{DO-LO}^{22} &= \begin{cases} 1 & \text{if} \quad \theta_2^I \leq \overline{\theta} \\ \widetilde{\theta}_{LO-DO} & \text{if} \quad \text{otherwise.} \end{cases} \end{aligned} \tag{II.16}$$

Case A.3: $\kappa_{MO-LO}^I \tau \leq \sigma$ and $PIN < \psi_{LO-NT}^U$

In this case we start with a market in which the equilibrium is $\mathcal{E}_1^{ND}$, where conditions ($II.1$) and ($II.2$) are satisfied. In this case, when we add the DP out of the 6 equilibria there are only two possible equilibria that satisfy these conditions: $\mathcal{E}_1^D$ and $\mathcal{E}_5^D$.[3] From Lemma C.2 we can see that

[3]It can be shown that when the conditions for the equilibrium $\mathcal{E}_1^{ND}$ are satisfied, the equilibrium $\mathcal{E}_3^D$ is not feasible.

$\mathcal{E}_1^D$ is an equilibrium if conditions ($II.8$) are satisfied. Similarly, $\mathcal{E}_5^D$ is an equilibrium if conditions ($II.13$) are satisfied, and in this case they can be rewritten as

$$\begin{aligned} & \theta_1^I > \max\{\overline{\theta}_{MO-DO}, \overline{\theta}_{LO-DO}\} \text{ and } \theta_2^I \leq \min\left\{\underline{\theta}, \theta_1^I\right\}, \\ \text{or} \quad & \widetilde{\theta}_{LO-DO} < \theta_1^I \text{ and } \underline{\theta} < \theta_2^I \leq \theta_1^I. \end{aligned}$$

Given that $\kappa_{MO-LO}^I \tau \leq \sigma$, we have that informed traders prefer MO to LO. Consequently, $\overline{\theta}_{MO-DO} \geq \overline{\theta}_{LO-DO}$ and, therefore, we have that $\mathcal{E}_5^D$ is an equilibrium if

$$\begin{aligned} & \theta_1^I > \overline{\theta}_{MO-DO} \text{ and } \theta_2^I \leq \min\left\{\underline{\theta}, \theta_1^I\right\}, \\ \text{or} \quad & \widetilde{\theta}_{LO-DO} < \theta_1^I \text{ and } \underline{\theta} < \theta_2^I \leq \theta_1^I. \end{aligned}$$

As a result in this case the optimal strategy profiles of a trader at $t = 1$ are:

$$\begin{cases} (BMO, SMO, BLO, BLO) & \text{if} \quad \theta_1^I \leq \widehat{\theta}_{MO-DO} \\ (BDO, SDO, BLO, BLO) & \text{if} \quad \theta_1^I > \theta_{DO-LO}^{21}, \end{cases}$$

where

$$\theta_{DO-LO}^{21} = \begin{cases} \overline{\theta}_{MO-DO} & \text{if} \quad \theta_2^I \leq \min\left\{\underline{\theta}, \theta_1^I\right\} \\ \widetilde{\theta}_{LO-DO} & \quad \underline{\theta} < \theta_2^I \leq \theta_1^I. \end{cases} \tag{II.17}$$

Case A.4: $\kappa_{MO-LO}^I \tau \leq \sigma$ and $PIN \geq \psi_{LO-NT}^U$

In this case we start with a market in which the equilibrium is $\mathcal{E}_2^{ND}$, where conditions (**??**) and (**??**) are satisfied. In this case, when we add the DP out of the 6 equilibria there are only three possible equilibria that satisfy these conditions: $\mathcal{E}_2^D$, $\mathcal{E}_5^D$, and $\mathcal{E}_6^D$.[4] From Lemma C.2 we can see that $\mathcal{E}_2^D$ is an equilibrium if conditions ($II.10$) are satisfied. Similarly, $\mathcal{E}_5^D$ is an equilibrium if conditions ($II.13$) are satisfied, and in this case they can be rewritten as

$$\widetilde{\theta}_{LO-DO} < \theta_1^I \text{ and } \overline{\theta} < \theta_2^I \leq \theta_1^I.$$

Finally, $\mathcal{E}_6^D$ is an equilibrium if conditions ($II.14$) are satisfied, and they can be rewritten as

$$\begin{aligned} & \theta_1^I > \max\{\overline{\theta}_{MO-DO}, \overline{\theta}_{LO-DO}\} \text{ and } \theta_2^I \leq \min\left\{\underline{\theta}, \theta_1^I\right\}, \\ \text{or} \quad & \widetilde{\theta}_{LO-DO} < \theta_1^I \text{ and } \underline{\theta} < \theta_2^I \leq \min\left\{\overline{\theta}, \theta_1^I\right\}. \end{aligned}$$

Given that $\kappa_{MO-LO}^I \tau \leq \sigma$, it follows that informed prefer MO to LO and, therefore, $\overline{\theta}_{LO-DO} \leq \overline{\theta}_{MO-DO}$, which implies $\max\{\overline{\theta}_{MO-DO}, \overline{\theta}_{LO-DO}\} = \overline{\theta}_{MO-DO}$. Hence, we can rewrite them as

$$\begin{aligned} & \theta_1^I > \overline{\theta}_{MO-DO} \text{ and } \theta_2^I \leq \min\left\{\underline{\theta}, \theta_1^I\right\}, \\ \text{or} \quad & \widetilde{\theta}_{LO-DO} < \theta_1^I \text{ and } \underline{\theta} < \theta_2^I \leq \min\left\{\overline{\theta}, \theta_1^I\right\}. \end{aligned}$$

[4] It can be shown that when the conditions for the equilibrium $\mathcal{E}_2^{ND}$ are satisfied, the equilibria $\mathcal{E}_3^D$ and $\mathcal{E}_4^D$ are not feasible.

As a result, the optimal strategy profiles of a trader at $t = 1$ are

$$\begin{aligned}
&\begin{cases} (BMO, SMO, NT, NT) & \text{if} \quad \theta_1^I \leq \overline{\theta}_{MO-DO} \\ (BDO, SDO, NT, NT) & \text{if} \quad \theta^3_{DO-NT} < \theta_1^I \leq \theta^3_{DO-LO} \\ (BDO, SDO, BLO, BLO) & \text{if} \quad \theta_1^I > \theta^3_{DO-LO}, \end{cases} \\
\text{where } \theta^3_{DO-NT} &= \begin{cases} \overline{\theta}_{MO-DO} & \text{if} \quad \theta_2^I \leq \min\{\underline{\theta}, \theta_1^I\} \\ \widetilde{\theta}_{LO-DO} & \quad \underline{\theta} < \theta_2^I \leq \min\{\overline{\theta}, \theta_1^I\} \\ 1 & \quad \min\{\overline{\theta}, \theta_1^I\} < \theta_2^I, \end{cases} \qquad \text{(II.18)} \\
\theta^3_{DO-LO} &= \begin{cases} 1 & \text{if} \quad \theta_2^I \leq \overline{\theta} \\ \widetilde{\theta}_{LO-DO} & \text{if} \quad \overline{\theta} < \theta_2^I \leq \theta_1^I. \end{cases}
\end{aligned}$$

Case B. From Lemma C.2, we know that in this case there are only two possible equilibria when there is access to the DP: $\mathcal{E}_2^D$ and $\mathcal{E}_6^D$. Moreover, we have that $\mathcal{E}_2^D$ is an equilibrium if conditions ($II.10$) are satisfied, while $\mathcal{E}_6^D$ is an equilibrium if

$$\begin{aligned}
&\theta_1^I > \max\{\overline{\theta}_{MO-DO}, \overline{\theta}_{LO-DO}\} \text{ and } \theta_2^I \leq \min\{\underline{\theta}, \theta_1^I\}, \\
\text{or} \quad &\widetilde{\theta}_{LO-DO} < \theta_1^I, \text{ and } \underline{\theta} < \theta_2^I \leq \theta_1^I.
\end{aligned}$$

Given that $\kappa^I_{MO-LO}\tau < \sigma$, it follows that informed prefer MO to LO and, therefore, $\overline{\theta}_{LO-DO} < \overline{\theta}_{MO-DO}$, which implies $\max\{\overline{\theta}_{MO-DO}, \overline{\theta}_{LO-DO}\} = \overline{\theta}_{MO-DO}$. Hence, we can rewrite them as

$$\begin{aligned}
&\theta_1^I > \overline{\theta}_{MO-DO} \text{ and } \theta_2^I \leq \min\{\underline{\theta}, \theta_1^I\}, \\
\text{or} \quad &\widetilde{\theta}_{LO-DO} < \theta_1^I \text{ and } \underline{\theta} < \theta_2^I \leq \theta_1^I.
\end{aligned}$$

As a result, the optimal strategy profiles of a trader at $t = 1$ are

$$\begin{aligned}
&\begin{cases} (BMO, SMO, NT, NT) & \text{if} \quad \theta_1^I \leq \overline{\theta}_{MO-DO} \\ (BDO, SDO, NT, NT) & \text{if} \quad \widehat{\theta}^3_{DO-NT} < \theta_1^I, \end{cases} \\
\text{where } \widehat{\theta}^3_{DO-NT} &= \begin{cases} \overline{\theta}_{MO-DO} & \text{if} \quad \theta_2^I \leq \min\{\underline{\theta}, \theta_1^I\} \\ \widetilde{\theta}_{LO-DO} & \text{if} \quad \underline{\theta} < \theta_2^I \leq \theta_1^I. \end{cases}
\end{aligned}$$

■

Internet Appendix III (Proof of Propositions 3, 4 and 5)

Proof of Proposition 3. Let us denote by $I_t^{ND,i}$ ($I_t^{D,i}$) the price informativeness in trading period t corresponding to the equilibrium $\mathcal{E}_i^{ND}$ ($\mathcal{E}_i^{D}$). Note that

$$\begin{aligned} I_1^{ND,i} &= \mathbb{E}\left(Var\left(V\right) - Var\left(V|\left(A_2^{ND,i}, B_2^{ND,i}\right)\right)\right) \text{ and} \\ I_2^{ND,i} &= \mathbb{E}\left(Var\left(V\right) - Var\left(V|\left(A_2^{ND,i}, B_2^{ND,i}\right)\right), \left(A_3^{ND,i}, B_3^{ND,i}\right)\right), \end{aligned}$$

where $\left(A_2^{ND,i}, B_2^{ND,i}\right)$ and $\left(A_3^{ND,i}, B_3^{ND,i}\right)$ represent the best prices at the end of period 1 and at the end of period 2, respectively, corresponding to the equilibrium $\mathcal{E}_i^{ND}$. Analogously, we can define $I_1^{D,i}$ and $I_2^{D,i}$.

We first compute the price informativeness at $t = 1$ for the equilibria of the single-venue market. Next, we use the following expressions:

$$Var\left(V\right) = \kappa^2\tau^2,$$

$$\begin{aligned} Var\left(V|\left(A_2^{ND,i}, B_2^{ND,i}\right)\right) &= pr\left(V = V^H|\left(A_2^{ND,i}, B_2^{ND,i}\right)\right)\left(V^H - \mathbb{E}\left(V|\left(A_2^{ND,i}, B_2^{ND,i}\right)\right)\right)^2 \\ &\quad + pr\left(V = V^L|\left(A_2^{ND,i}, B_2^{ND,i}\right)\right)\left(V^L - \mathbb{E}\left(V|\left(A_2^{ND,i}, B_2^{ND,i}\right)\right)\right)^2, \end{aligned}$$

where pr refers to the probability and

$$\begin{aligned} pr\left(V = V^H|\left(A_2^{ND,i}, B_2^{ND,i}\right)\right) &= \frac{pr\left(\left(A_2^{ND,i}, B_2^{ND,i}\right)|V = V^H\right)pr(V = V^H)}{pr\left(\left(A_2^{ND,i}, B_2^{ND,i}\right)\right)}, \\ pr\left(V = V^L|\left(A_2^{ND,i}, B_2^{ND,i}\right)\right) &= 1 - pr\left(V = V^H|\left(A_2^{ND,i}, B_2^{ND,i}\right)\right), \text{ and} \end{aligned}$$

$$\mathbb{E}\left(V|\left(A_2^{ND,i}, B_2^{ND,i}\right)\right) = pr\left(V = V^H|\left(A_2^{ND,i}, B_2^{ND,i}\right)\right)V^H + pr\left(V = V^L|\left(A_2^{ND,i}, B_2^{ND,i}\right)\right)V^L.$$

We compute these expressions for each equilibria of the single-venue market as follows:

$\mathcal{E}_1^{ND}$:

$\left(A_2^{ND,1}, B_2^{ND,1}\right)$	$pr\left(\left(A_2^{ND,1}, B_2^{ND,1}\right)\right)$	$pr\left(V = V^H\|\left(A_2^{ND,1}, B_2^{ND,1}\right)\right)$	$\mathbb{E}\left(V\|\left(A_2^{ND,1}, B_2^{ND,1}\right)\right)$
(A_1^2, B_1^1)	$\frac{\lambda\pi+1-\lambda}{2}$	$\frac{\lambda\pi+\frac{1-\lambda}{2}}{\lambda\pi+1-\lambda}$	$\mu + \frac{\lambda\pi}{\lambda\pi+1-\lambda}\kappa\tau$
$(A_1^1, B_1^1 + \tau)$	$\frac{\lambda(1-\pi)}{2}$	$\frac{1}{2}$	μ
(A_1^1, B_1^2)	$\frac{\lambda\pi+1-\lambda}{2}$	$\frac{\frac{1-\lambda}{2}}{\lambda\pi+1-\lambda}$	$\mu - \frac{\lambda\pi}{\lambda\pi+1-\lambda}\kappa\tau$
$(A_1^1 - \tau, B_1^1)$	$\frac{\lambda(1-\pi)}{2}$	$\frac{1}{2}$	μ

and

$\left(A_2^{ND,1}, B_2^{ND,1}\right)$	$Var\left(V \vert \left(A_2^{ND,1}, B_2^{ND,1}\right)\right)$
(A_1^2, B_1^1)	$\frac{(2\lambda\pi+1-\lambda)(1-\lambda)}{(\lambda\pi+1-\lambda)^2}\kappa^2\tau^2$
$(A_1^1, B_1^1+\tau)$	$\kappa^2\tau^2$
(A_1^1, B_1^2)	$\frac{(2\lambda\pi+1-\lambda)(1-\lambda)}{(\lambda\pi+1-\lambda)^2}\kappa^2\tau^2$
$(A_1^1-\tau, B_1^1)$	$\kappa^2\tau^2$

Using the previous tables, it follows that $I_1^{ND,1} = \frac{\lambda^2\pi^2}{(\lambda\pi+1-\lambda)}\kappa^2\tau^2$.

$\mathcal{E}_2^{ND}$:

$\left(A_2^{ND,2}, B_2^{ND,2}\right)$	$pr\left(\left(A_2^{ND,2}, B_2^{ND,2}\right)\right)$	$pr\left(V = V^H \vert \left(A_2^{ND,2}, B_2^{ND,2}\right)\right)$	$\mathbb{E}\left(V \vert \left(A_2^{ND,2}, B_2^{ND,2}\right)\right)$
(A_1^1, B_1^1)	$\lambda(1-\pi)$	$\frac{\lambda(1-\pi)\frac{1}{2}}{\lambda(1-\pi)} = \frac{1}{2}$	μ
(A_1^2, B_1^1)	$\frac{\lambda\pi+1-\lambda}{2}$	$\frac{\left(\lambda\pi+\frac{1-\lambda}{2}\right)\frac{1}{2}}{\frac{\lambda\pi+1-\lambda}{2}} = \frac{\lambda\pi+\frac{1-\lambda}{2}}{\lambda\pi+1-\lambda}$	$\mu + \frac{\lambda\pi}{\lambda\pi+1-\lambda}\kappa\tau$
(A_1^1, B_1^2)	$\frac{\lambda\pi+1-\lambda}{2}$	$\frac{\left(\frac{1-\lambda}{2}\right)\frac{1}{2}}{\frac{\lambda\pi+1-\lambda}{2}} = \frac{\frac{1-\lambda}{2}}{\lambda\pi+1-\lambda}$	$\mu - \frac{\lambda\pi}{\lambda\pi+1-\lambda}\kappa\tau$

and

$\left(A_2^{ND,2}, B_2^{ND,2}\right)$	$Var\left(V \vert \left(A_2^{ND,2}, B_2^{ND,2}\right)\right)$
(A_1^1, B_1^1)	$\kappa^2\tau^2$
(A_1^2, B_1^1)	$\frac{(2\lambda\pi+1-\lambda)(1-\lambda)}{(\lambda\pi+1-\lambda)^2}\kappa^2\tau^2$
(A_1^1, B_1^2)	$\frac{(2\lambda\pi+1-\lambda)(1-\lambda)}{(\lambda\pi+1-\lambda)^2}\kappa^2\tau^2$

Using the previous tables, it follows that $I_1^{ND,2} = \frac{\lambda^2\pi^2}{(\lambda\pi+1-\lambda)}\kappa^2\tau^2$.

$\mathcal{E}_3^{ND}$:

$\left(A_2^{ND,3}, B_2^{ND,3}\right)$	$pr\left(\left(A_2^{ND,3}, B_2^{ND,3}\right)\right)$	$pr\left(V = V^H \vert \left(A_2^{ND,3}, B_2^{ND,3}\right)\right)$	$\mathbb{E}\left(V \vert \left(A_2^{ND,3}, B_2^{ND,3}\right)\right)$
(A_1^2, B_1^1)	$\frac{1-\lambda}{2}$	$\frac{1}{2}$	μ
$(A_1^1, B_1^1+\tau)$	$\frac{\lambda}{2}$	$\frac{1+\pi}{2}$	$\mu + \pi\kappa\tau$
(A_1^1, B_1^2)	$\frac{1-\lambda}{2}$	$\frac{1}{2}$	μ
$(A_1^1-\tau, B_1^1)$	$\frac{\lambda}{2}$	$\frac{1-\pi}{2}$	$\mu - \pi\kappa\tau$

and

$\left(A_2^{ND,3}, B_2^{ND,3}\right)$	$Var\left(V \vert \left(A_2^{ND,3}, B_2^{ND,3}\right)\right)$
(A_1^2, B_1^1)	$\kappa^2\tau^2$
$(A_1^1, B_1^1+\tau)$	$\left(1-\pi^2\right)\kappa^2\tau^2$
(A_1^1, B_1^2)	$\kappa^2\tau^2$
$(A_1^1-\tau, B_1^1)$	$\left(1-\pi^2\right)\kappa^2\tau^2$

Using the previous tables, it follows that $I_1^{ND,3} = \lambda\pi^2\kappa^2\tau^2$.

$\mathcal{E}_4^{ND}$:

$\left(A_2^{ND,4}, B_2^{ND,4}\right)$	$pr\left(\left(A_2^{ND,4}, B_2^{ND,4}\right)\right)$	$pr\left(V = V^H \mid \left(A_2^{ND,4}, B_2^{ND,4}\right)\right)$	$\mathbb{E}\left(V \mid \left(A_2^{ND,4}, B_2^{ND,4}\right)\right)$
(A_1^1, B_1^1)	$\lambda(1-\pi)$	$\frac{1}{2}$	μ
(A_1^2, B_1^1)	$\frac{1-\lambda}{2}$	$\frac{1}{2}$	μ
$(A_1^1, B_1^1+\tau)$	$\frac{\lambda\pi}{2}$	1	$\mu+\kappa\tau$
(A_1^1, B_1^2)	$\frac{1-\lambda}{2}$	$\frac{1}{2}$	μ
$(A_1^1-\tau, B_1^1)$	$\frac{\lambda\pi}{2}$	0	$\mu-\kappa\tau$

and

$\left(A_2^{ND,4}, B_2^{ND,4}\right)$	$Var\left(V \mid \left(A_2^{ND,4}, B_2^{ND,4}\right)\right)$
(A_1^1, B_1^1)	$\kappa^2\tau^2$
(A_1^2, B_1^1)	$\kappa^2\tau^2$
$(A_1^1, B_1^1+\tau)$	0
(A_1^1, B_1^2)	$\kappa^2\tau^2$
$(A_1^1-\tau, B_1^1)$	0

Using the previous tables, it follows that $I_1^{ND,4} = \lambda\pi\kappa^2\tau^2$.

In the two-venue market, if there is no order migration towards to the DP at $t=1$, then the price informativeness remains unchanged, i.e., $I_1^{D,i} = I_1^{ND,i}$, for all $i = 1, ..., 4$. For the two remaining equilibria, we have the following:

$\mathcal{E}_5^{D}$:

$\left(A_2^{D,5}, B_2^{D,5}\right)$	$pr\left(\left(A_2^{D,5}, B_2^{D,5}\right)\right)$	$pr\left(V = V^H \mid \left(A_2^{D,5}, B_2^{D,5}\right)\right)$	$\mathbb{E}\left(V \mid \left(A_2^{D,5}, B_2^{D,5}\right)\right)$
(A_1^1, B_1^1)	$\lambda\pi$	$\frac{1}{2}$	μ
(A_1^2, B_1^1)	$\frac{1-\lambda}{2}$	$\frac{1}{2}$	μ
$(A_1^1, B_1^1+\tau)$	$\frac{\lambda(1-\pi)}{2}$	$\frac{1}{2}$	μ
(A_1^1, B_1^2)	$\frac{1-\lambda}{2}$	$\frac{1}{2}$	μ
$(A_1^1-\tau, B_1^1)$	$\frac{\lambda(1-\pi)}{2}$	$\frac{1}{2}$	μ

In this case, $Var\left(V \mid \left(A_2^{D,5}, B_2^{D,5}\right)\right) = Var(V)$, for all $\left(A_2^{D,5}, B_2^{D,5}\right)$. This implies that $I_1^{D,5} = 0$.

$\mathcal{E}_6^D$:

$\left(A_2^{D,6}, B_2^{D,6}\right)$	$pr\left(\left(A_2^{D,6}, B_2^{D,2}\right)\right)$	$pr\left(V = V^H \mid \left(A_2^{D,6}, B_2^{D,6}\right)\right)$	$\mathbb{E}\left(V \mid \left(A_2^{D,6}, B_2^{D,6}\right)\right)$
(A_1^1, B_1^1)	$\lambda\pi + \lambda(1-\pi) = \lambda$	$\frac{\lambda\frac{1}{2}}{\lambda} = \frac{1}{2}$	μ
(A_1^2, B_1^1)	$\frac{1-\lambda}{2}$	$\frac{\left(\frac{1-\lambda}{2}\right)\frac{1}{2}}{\frac{1-\lambda}{2}} = \frac{1}{2}$	μ
(A_1^1, B_1^2)	$\frac{1-\lambda}{2}$	$\frac{\left(\frac{1-\lambda}{2}\right)\frac{1}{2}}{\frac{1-\lambda}{2}} = \frac{1}{2}$	μ

In this case, $Var\left(V \mid \left(A_2^{D,6}, B_2^{D,6}\right)\right) = Var(V)$, for all $\left(A_2^{D,6}, B_2^{D,6}\right)$. This implies that $I_1^{D,6} = 0$.

Finally, the comparison of price informativeness at $t = 1$ is summarized in the following table:

Price Informativeness at $t = 1$	$\mathcal{E}_1^D$	$\mathcal{E}_2^D$	$\mathcal{E}_3^D$	$\mathcal{E}_4^D$	$\mathcal{E}_5^D$	$\mathcal{E}_6^D$
$\mathcal{E}_1^{ND}$	$=$				$>$	
$\mathcal{E}_2^{ND}$		$=$			$>$	$>$
$\mathcal{E}_3^{ND}$			$=$		$>$	
$\mathcal{E}_4^{ND}$				$=$	$>$	$>$

■

Proof of Proposition 4. We first compute the ex-ante expected inside spread in the exchange for the equilibria in the single-venue market. In the following table, for each couple of best prices of the LOB at the end of $t = 1$, it is displayed the value of inside spread, S_1, and its corresponding probability for each equilibrium

	S_1	$prob^{\mathcal{E}_1^{ND}}$	$prob^{\mathcal{E}_2^{ND}}$	$prob^{\mathcal{E}_3^{ND}}$	$prob^{\mathcal{E}_4^{ND}}$
(A_1^1, B_1^1)	$2k_1\tau$	0	$\lambda(1-\pi)$	0	$\lambda(1-\pi)$
(A_1^2, B_1^1)	$(k_2 + k_1)\tau$	$\frac{\lambda\pi}{2} + \frac{1-\lambda}{2}$	$\frac{\lambda\pi}{2} + \frac{1-\lambda}{2}$	$\frac{1-\lambda}{2}$	$\frac{1-\lambda}{2}$
$(A_1^1, B_1^1 + \tau)$	$(2k_1 - 1)\tau$	$\frac{\lambda(1-\pi)}{2}$	0	$\frac{\lambda}{2}$	$\frac{\lambda\pi}{2}$
(A_1^1, B_1^2)	$(k_2 + k_1)\tau$	$\frac{\lambda\pi}{2} + \frac{1-\lambda}{2}$	$\frac{\lambda\pi}{2} + \frac{1-\lambda}{2}$	$\frac{1-\lambda}{2}$	$\frac{1-\lambda}{2}$
$(A_1^1 - \tau, B_1^1)$	$(2k_1 - 1)\tau$	$\frac{\lambda(1-\pi)}{2}$	0	$\frac{\lambda}{2}$	$\frac{\lambda\pi}{2}$

Using the values included in the previous table, it follows that the expected spreads at $t = 1$ in each equilibrium equal

$$
\begin{aligned}
\mathbb{E}_0\left(S_1^{\mathcal{E}_1^{ND}}\right) &= (2k_1 + (1 - \lambda(1-\pi))(k_2 - k_1)) - \lambda(1-\pi))\tau, \\
\mathbb{E}_0\left(S_1^{\mathcal{E}_2^{ND}}\right) &= (2k_1 + (1 - \lambda(1-\pi))(k_2 - k_1))\tau, \\
\mathbb{E}_0\left(S_1^{\mathcal{E}_3^{ND}}\right) &= (2k_1 + (1-\lambda)(k_2 - k_1) - \lambda)\tau, \text{ and} \\
\mathbb{E}_0\left(S_1^{\mathcal{E}_4^{ND}}\right) &= (2k_1 + (1-\lambda)(k_2 - k_1) - \lambda\pi)\tau.
\end{aligned}
$$

With access to the DP, if there is no order migration towards to the DP at $t = 1$, then the ex-ante inside spread remains unchanged, i.e., $\mathbb{E}_0\left(S_1^{\mathcal{E}_i^D}\right) = \mathbb{E}_0\left(S_1^{\mathcal{E}_i^{ND}}\right)$, for all $i = 1, ..., 4$. For the two remaining equilibria, the following table shows the value of inside spread and its corresponding probability for each couple of best prices:

	S_1	$prob^{\mathcal{E}_5^D}$	$prob^{\mathcal{E}_6^D}$
(A_1^1, B_1^1)	$2k_1\tau$	$\lambda\pi$	λ
(A_1^2, B_1^1)	$(k_2+k_1)\tau$	$\frac{1-\lambda}{2}$	$\frac{1-\lambda}{2}$
$(A_1^1, B_1^1+\tau)$	$(2k_1-1)\tau$	$\frac{\lambda(1-\pi)}{2}$	0
(A_1^1, B_1^2)	$(k_2+k_1)\tau$	$\frac{1-\lambda}{2}$	$\frac{1-\lambda}{2}$
$(A_1^1-\tau, B_1^1)$	$(2k_1-1)\tau$	$\frac{\lambda(1-\pi)}{2}$	0

From the previous table, it follows that

$$\begin{aligned}\mathbb{E}_0\left(S_1^{\mathcal{E}_5^D}\right) &= \left(2k_1+(1-\lambda)(k_2-k_1)-\lambda(1-\pi)\right)\tau \text{ and}\\ \mathbb{E}_0\left(S_1^{\mathcal{E}_6^D}\right) &= \left(2k_1+(1-\lambda)(k_2-k_1)\right)\tau.\end{aligned}$$

The change in inside spread due to the introduction of the DP is $\mathbb{E}_0\left(S_1^{\mathcal{E}_j^D}\right)-\mathbb{E}_0\left(S_1^{\mathcal{E}_i^{ND}}\right)$

$\mathbb{E}_0\left(S_1^{\mathcal{E}_j^D}\right)-\mathbb{E}_0\left(S_1^{\mathcal{E}_i^{ND}}\right)$						
	$\mathcal{E}_1^D$	$\mathcal{E}_2^D$	$\mathcal{E}_3^D$	$\mathcal{E}_4^D$	$\mathcal{E}_5^D$	$\mathcal{E}_6^D$
$\mathcal{E}_1^{ND}$	0				$-\lambda\pi\left(k_2-k_1\right)\tau$	
$\mathcal{E}_2^{ND}$		0			$-\lambda\left(\pi(k_2-k_1-1)+1\right)\tau$	$-\lambda\pi\left(k_2-k_1\right)\tau$
$\mathcal{E}_3^{ND}$			0		$\lambda\pi\tau$	
$\mathcal{E}_4^{ND}$				0	$\lambda\left(2\pi-1\right)\tau$	$\lambda\pi\tau$

Therefore, at $t=1$ we have the following relationship between spreads in the two-venue market and the single-venue market.

$\mathbb{E}_0\left(S_1\right)$	$\mathcal{E}_1^D$	$\mathcal{E}_2^D$	$\mathcal{E}_3^D$	$\mathcal{E}_4^D$	$\mathcal{E}_5^D$	$\mathcal{E}_6^D$
$\mathcal{E}_1^{ND}$	$=$				$>$	
$\mathcal{E}_2^{ND}$		$=$			$>$	$>$
$\mathcal{E}_3^{ND}$			$=$		$<$	
$\mathcal{E}_4^{ND}$				$=$	$<$	$<$

■

Proof of Proposition 5. We first summarize the expected profits for rational traders at $t=1$ in the single-venue market:

	Expected Profits at $t=1$	
	Informed Trader	Uninformed Trader
$\mathcal{E}_1^{ND}$	$(\kappa - k_1)\,\tau$	$\frac{\delta}{2}\left((\lambda\pi + 1 - \lambda)(k_1 - 1) - \lambda\pi\kappa\right)\tau$
$\mathcal{E}_2^{ND}$	$(\kappa - k_1)\,\tau$	0
$\mathcal{E}_3^{ND}$	$\frac{\delta(1-\lambda)}{2}(\kappa + k_1 - 1)\,\tau$	$\frac{\delta}{2}\left((\lambda\pi + 1 - \lambda)(k_1 - 1) - \lambda\pi\kappa\right)\tau$
$\mathcal{E}_4^{ND}$	$\frac{\delta(1-\lambda)}{2}(\kappa + k_1 - 1)\,\tau$	0

When traders have access to the DP, the unconditional expected profits of investors for the first four equilibria coincide with the corresponding ones given in the previous table. For the two remaining equilibria, we have the following unconditional expected profits:

Equilibrium	Expected Profits of an Informed Trader at $t=1$
$\mathcal{E}_5^D$	$\theta_1^I\kappa\tau + (1-\theta_1^I)\delta\left(\kappa - k_1 - (k_2 - k_1)\left(\lambda\pi + \frac{1-\lambda}{2}\right)\right)\tau$ if $\theta_2^I \leq \frac{\kappa - k_1}{\kappa}$ $\theta_1^I\kappa\tau + (1-\theta_1^I)\delta\left(\kappa - k_1 - (k_2 - k_1)\frac{1-\lambda}{2}\right)\tau$ if $\theta_2^I > \frac{\kappa - k_1}{\kappa}$
$\mathcal{E}_6^D$	$\theta_1^I\kappa\tau + (1-\theta_1^I)\delta\left(\kappa - k_1 - (k_2 - k_1)\left(\lambda\pi + \frac{1-\lambda}{2}\right)\right)\tau$ if $\theta_2^I \leq \frac{\kappa - k_1}{\kappa}$ $\theta_1^I\kappa\tau + (1-\theta_1^I)\delta\left(\kappa - k_1 - (k_2 - k_1)\frac{1-\lambda}{2}\right)\tau$ if $\theta_2^I > \frac{\kappa - k_1}{\kappa}$

and

Expected Profits at $t=1$	
Equilibrium	Uninformed Trader
$\mathcal{E}_5^D$	$\frac{\delta}{2}\left((\lambda\pi + 1 - \lambda)(k_1 - 1) - \lambda\pi\kappa\right)\tau$ if $\theta_2^I \leq \dfrac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}}$ $\frac{\delta}{2}(1-\lambda)(k_1 - 1)\tau$ if $\theta_2^I > \frac{\kappa - k_1 + 1}{\kappa + \frac{1}{2}}$
$\mathcal{E}_6^D$	0

The following tables show how the unconditional expected profits of rational traders change due to the introduction of the DP at $t=1$:

Informed Trader	$\mathcal{E}_1^D$	$\mathcal{E}_2^D$	$\mathcal{E}_3^D$	$\mathcal{E}_4^D$	$\mathcal{E}_5^D$	$\mathcal{E}_6^D$
$\mathcal{E}_1^{ND}$	$=$				$<$	
$\mathcal{E}_2^{ND}$		$=$			$<$	$<$
$\mathcal{E}_3^{ND}$			$=$		$<$	
$\mathcal{E}_4^{ND}$				$=$	$<$	$<$

and

Uninformed Trader	$\mathcal{E}_1^D$	$\mathcal{E}_2^D$	$\mathcal{E}_3^D$	$\mathcal{E}_4^D$	$\mathcal{E}_5^D$	$\mathcal{E}_6^D$
$\mathcal{E}_1^{ND}$	$=$				$\leq$	
$\mathcal{E}_2^{ND}$		$=$			$<$	$=$
$\mathcal{E}_3^{ND}$			$=$		$\leq$	
$\mathcal{E}_4^{ND}$				$=$	$<$	$=$

■

Internet Appendix IV (Additional Graphs)

In this Appendix we illustrate graphically how different parameter values affect the existence of the equilibria. In Figure 3, in the paper, we depicted the optimal strategies at $t = 1$ in the single-venue market for $k_1 = 5$. In what it follows, we present two additional cases: when the market liquidity is high, $k_1 = 2$, and very illiquid $k_1 = 30$. This last example, with values that are not very realistic is selected to display the four possible equilibria. Remember that in the case $k_1 = 1$ the only possible equilibrium is (BMO, SMO, NT, NT).

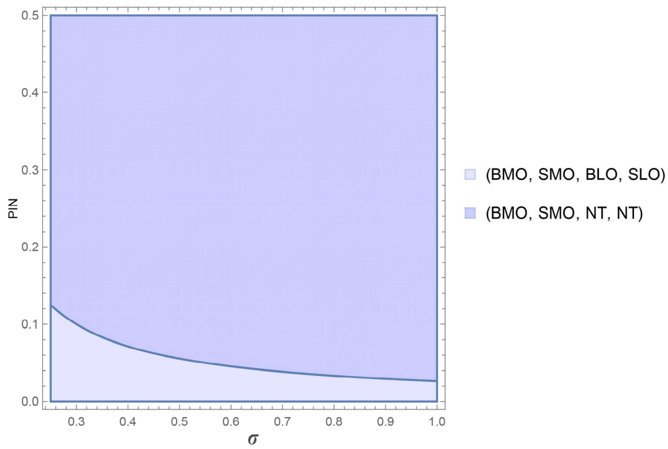

Figure IV.1: Optimal strategies at $t = 1$ in the single-venue market. Parameters values: $k_1 = 2$, $\lambda = 0.5$, $\tau = 0.05$, $\delta = 0.95$.

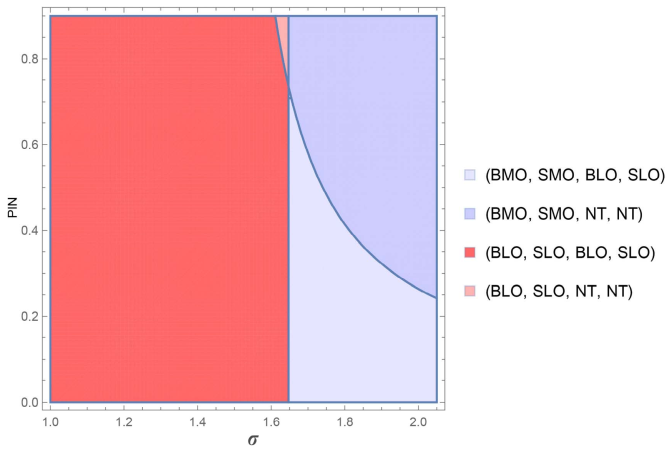

Figure IV.2: Optimal strategies at $t = 1$ in the single-venue market. Parameters values: $k_1 = 30$, $\lambda = 0.9$, $\tau = 0.05$, $\delta = 0.95$.

Figure IV.3 illustrates the optimal strategies at $t = 1$ with respect to the fundamental asset's volatility and information asymmetry in the case market is very liquid, $k_1 = 2$, and for several values of $\theta_1^I \in \{0.05, 0.19, 0.25, 0.35\}$, which we show in Panels a), b), c), and d), respectively.

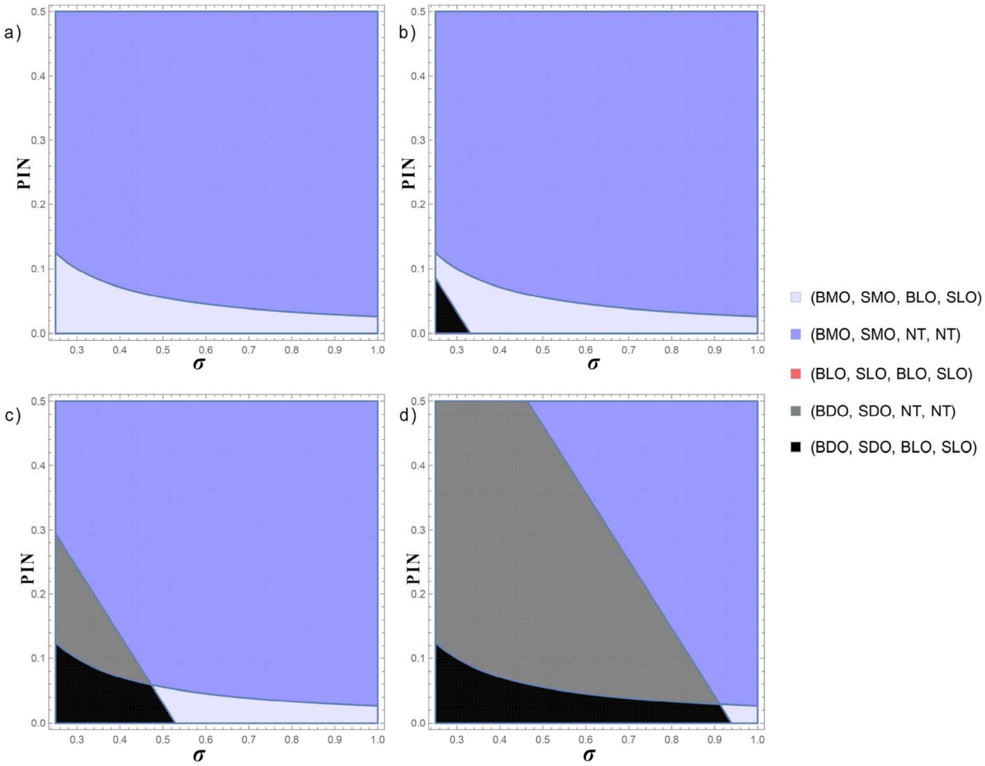

Figure IV.3: Optimal strategies at $t = 1$ with dark pool. Parameters values: $k_1 = 2$, $k_2 = 3$, $\lambda = 0.5$, $\tau = 0.05$, and $\delta = 0.95$. In Panel a) $\theta_1^I = 0.05$, in Panel b) $\theta_1^I = 0.19$, in Panel c) $\theta_1^I = 0.25$, and in Panel d) $\theta_1^I = 0.35$.

Figure IV.4 illustrates the optimal strategy profiles at $t = 1$ with respect to the asset's volatility and the probability of execution for the informed trader in the DP in the first trading period (θ_1^I) in the case market is very liquid, $k_1 = 2$.

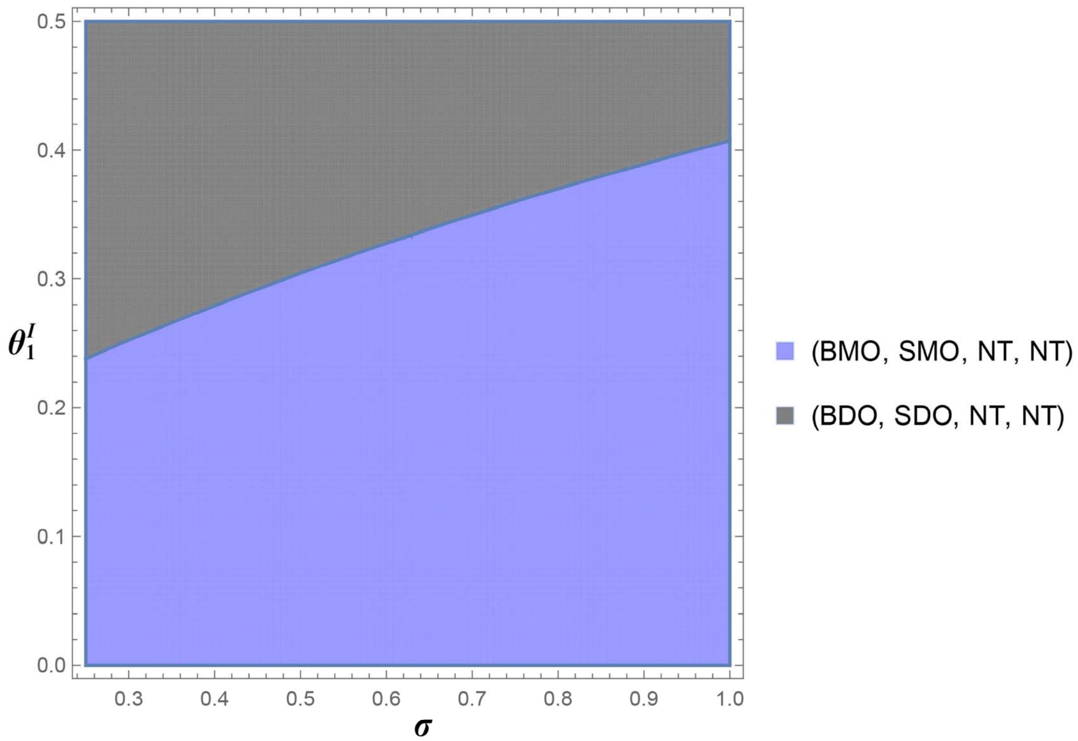

Figure IV.4: Optimal strategies at $t = 1$ with dark pool. Parameters values: $k_1 = 2$, $k_2 = 3$, $\lambda = 0.5$, $\pi = 0.5$, $\tau = 0.05$, and $\delta = 0.95$.