# Information and Optimal Trading Strategies with Dark Pools *

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**Abstract**

We examine the competition between a transparent exchange organized as a limit order book and an opaque dark pool in the presence of asymmetric information. We show that the coexistence of a dark pool with an exchange not only enlarges traders' strategy set, but may also induce trading venue substitution, a change in the order type, and an increase in market participation. Consequently, dark trading affects market quality and traders' profits. These effects depend on stock market characteristics (fundamental volatility, liquidity, and adverse selection) and traders' characteristics (immediacy and information). We derive new empirical and policy implications from our analysis.

*Keywords*: trading venues, dark liquidity, limit order book, price risk, adverse selection, double volume cap

*JEL codes*: G12, G14, G18

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# Introduction

In today's financial markets, traders have access to competing trading venues with different levels of transparency. In addition to traditional exchanges (also known as lit markets), market participants can trade in dark pools (opaque trading venues). As of August 2020, trading volume in dark pools accounted for 12.31% market share of consolidated share volume in US markets and 6.32% of consolidated value traded in European markets.[1] The growing importance of dark pools as alternative trading venues, and the segmentation of the order flow into lit and dark venues has led regulators to focus on the impact of dark trading on market quality. Thus, in 2018, a regulatory reform in Europe, MiFID II, was implemented to offer greater protection for investors and foster transparency in financial markets. At the same time, in 2018 the U.S. Securities and Exchange Commission (SEC) also adopted amendments to Regulation ATS (Alternative Trading Systems) to enhance operational transparency and regulatory oversight. Both reforms aim to limit dark trading and its potential negative impact on market quality.

Limit order books have replaced most of the traditional exchanges and compete for order flow with dark pools. Recent empirical and experimental work emphasizes the important role of information in the segmentation of the order flow and price discovery process when multiple trading venues compete (Ready, 2014; Comerton-Forde and Putniņš, 2015; Hatheway et al., 2017; Brogaard and Pan, 2019; Hendershott et al., 2020). The mixed results found in this literature and the regulators' concerns that dark trading harms price discovery suggest that there is a need for a new theoretical framework to understand the price discovery and liquidity provision of markets in which an exchange organized as a limit order book competes with a dark pool. As information asymmetry plays a fundamental role in the price discovery process, a better understanding of the competition between a limit order book and a dark pool in the presence of asymmetric information is imperative.[2]

In this paper, we examine how dark trading affects price discovery and market performance in a two-period model. We show that the effects of competition between a dark pool and a transparent exchange depend on the stock market and trader characteristics. For example, in the first trading period, for high fundamental volatility stocks, price informativeness decreases, liquidity increases, while

[1] See Rosenblatt Securities, "Let there be light," August 2020, and "Let there be light, European edition," August 2020.

[2] Bhattacharya and Saar (2020), Brolley and Malinova (2020), and Riccó et al. (2020) explain that the role of information on the order selection decision is not well understood theoretically, and it is methodologically challenging. Zhu (2014) also points out that the competition between an exchange that is organized as a limit order book and a dark pool is complex and difficult to solve analytically.

trading volume in the exchange decreases and there is trade destruction. However, for low fundamental volatility stocks price informativeness and liquidity decrease, while volume in the exchange remains the same and there is trade creation. In the second trading period we obtain ambiguous results, showing that market conditions as a whole are relevant in such a comparison. Interestingly, we find that the effects in the two periods may differ, although investors behave similarly. This is in contrast to other theoretical frameworks, and is due to the fact that traders learn from the prices in the limit order book and optimally change their trading strategies accordingly. Our analysis reconciles conflicting theoretical results and empirical evidence on the effects of dark pools on market quality. Moreover, we provide implications for the current policy debate regarding the regulation of dark pools as our results point out that regulators should be cautious in choosing a "one-size-fits-all" regulatory policy, given potential unintended consequences.

Our model reflects the main characteristics of today's financial markets. The exchange is organized as a fully transparent limit order book that competes for order flow with a dark pool. Although many types of dark pools exist, our model makes two assumptions that capture their main features: (1) no pre-trade transparency because dark pools are completely opaque in the sense that they do not display available liquidity, which makes execution uncertain; and (2) dark pools do not determine prices and derive them from those prevailing in the exchange as the midpoint between the best bid and ask prices at any point in time. This type of pricing is typical of dark pools owned by agency brokers or exchanges.[3]

Asymmetric information plays a crucial role in the decision to provide or demand liquidity in the limit order book or whether to submit an order to the dark pool. The segmentation of the order flow alters the revelation of information in prices and therefore it may affect rational traders' optimal strategies. In order to understand the effect of long-lived asymmetric information on the optimal traders' strategies in our framework, we build a two-period trading model. In each trading period, a new trader arrives to the market and may trade one unit of a risky asset. There are two possible types of traders. On the one hand, there are rational traders who strategically choose whether or not to trade, and if they trade, they simultaneously select the venue and the type of order that maximize profits given their information. Rational traders can submit several order types: a market order or limit order to the exchange, or a dark pool order. In addition, rational traders may be informed if

[3]Some dark pools offer other types of price improvements that are not necessarily equal to the midpoint (see, for example, Brolley 2020).

they know the liquidation value of the asset perfectly, or (privately) uninformed if they know only the distribution of the liquidation value of the asset conditional on public information. A unique feature of our model with multiple venues is that, since the limit order book is transparent, an uninformed trader may learn about the liquidation value of the asset from the changes of prices in the limit order book. On the other hand, there are liquidity traders who participate in the market for liquidity reasons and submit market orders only to the exchange to ensure immediate execution.

We compare traders' equilibrium strategies and market quality indicators in a model in which traders are restricted to trade only in the exchange (we refer to this setup as the single-venue market model) to one in which agents have access to a dark pool and an exchange (we refer to this setup as the two-venue market model). In the first trading period of the single-venue market model, we show that when the fundamental asset's volatility is above a cut-off value (high fundamental volatility stocks), and therefore, an informed trader has a large informational advantage, he chooses to demand liquidity, i.e., to place a market order, because the price improvement of a limit order is not sufficient to compensate for its execution risk. Otherwise, that is, if stock fundamental volatility is low, an informed trader supplies liquidity, i.e., he places a limit order. By contrast, it is optimal for an uninformed trader to choose between a limit order and not trading depending on the degree of adverse selection. An uninformed trader selects a limit order when there is a high probability that the order will be executed against the order of a liquidity trader (instead of an informed trader) or when the fundamental volatility of the asset, and hence, the informational advantage of an informed trader is low enough. In the second trading period of the single-venue market model, an informed trader always wants to trade due to his informational advantage over the other traders. In addition, the uninformed trader chooses between demanding liquidity or not trading, and the choice depends on the value of the asset inferred from the limit order book compared to the best prices.

A unique feature of our two-venue model is that we allow dark orders to be first routed to the dark pool and then re-routed to the exchange. As a result, we are able to study an important trade-off that traders face when submitting a dark order: they improve the execution price but face the risk that, in case of non-execution in the dark and the order returning to the exchange, the price in the limit order book has moved against them. We show that traders' access to the dark pool may induce trading venue substitution, but also a change in order type in the exchange, and an increase in market participation compared to the single-venue market. In the first trading period in the two-venue market model, an informed trader finds dark orders more attractive than limit and market orders when the execution risk in the dark pool is sufficiently low. In contrast, an uninformed trader does not go to the dark pool in the first trading period since the price improvement is not sufficient to induce a change in trading venue. Nevertheless, the existence of a dark pool alongside the exchange may change the uninformed trader's optimal submission strategy from not trading to supplying liquidity in the exchange. This occurs when adverse selection in the exchange decreases because the informed trader migrates from the exchange to the dark pool. In such a case, there is order flow segmentation in the first trading period. In the second trading period, we find that informed and uninformed traders submit dark orders if the execution risk in the dark pool is low enough and the price improvement is significant. Whether order flow segmentation occurs or not in the second trading period depends crucially on the stock and trader characteristics.

We find that the impact of a dark pool that coexists with an exchange on market quality depends on the time period, type of order that migrates to the dark pool and trader that submitted it. In the first trading period, for high fundamental volatility stocks, price informativeness decreases, liquidity increases, while trading volume in the exchange decreases and there is trade destruction. However, for low fundamental volatility stocks price informativeness and liquidity decrease, while volume in the exchange remains the same and there is trade creation. For both types of stocks, the profits of rational traders are never lower in the two-venue market compared to the single-venue market. In the second trading period, we find that the informational content of prices is not necessarily lower in the two-venue market and that the expected profits of an uninformed trader may decline due to a deterioration in market quality.

Our work contributes to a growing body of theoretical research on the effects of competition between exchanges and dark pools. More specifically, we are the first to model the competition between an exchange organized as a limit order book and a dark pool in the presence of long-lived asymmetric information.[4] In a static set-up, Hendershott and Mendelson (2000) find that a crossing network (similar to a dark pool) that competes with a dealer market is characterized by positive liquidity externality and, at the same time, it generates a negative crowding externality, leading to ambiguous effects on market quality that depend on the insider's informational advantage. Degryse

[4] Glosten (1994), Chakravarty and Holden (1995), Seppi (1997), Biais et al. (2000), and Kaniel and Liu (2006) emphasize the role of asymmetric information in the choice of order submission strategies in a single trading venue. In addition, Parlour (1998), Foucault (1999), Parlour and Seppi (2003), Foucault et al. (2005), Goettler et al. (2009), Rosu (2009), Brolley and Malinova (2020), and Riccó et al. (2020) study the optimal choice of order type in dynamic models.

et al. (2009) show that the same positive and negative externalities remain in a dynamic setup and analyze how welfare and the order flow dynamics depend on the degree of market transparency.[5]

Our paper is more closely related to Zhu (2014), Buti et al. (2017) and Brolley (2020). Like in Zhu (2014), we examine the role of asymmetric information in competing trading venues. However, we model the competition of a dark pool with a limit order book; therefore, in our framework, traders can both demand liquidity (as in Zhu, 2014) and supply liquidity to the exchange. Moreover, we propose a two-period model, which allows for the first time to examine how information is gradually incorporated in the limit order book, and how traders' strategies reflect this change. Interestingly, under some parameter configurations, we find the same result as Zhu (2014) that dark pools improve price informativeness (when in the second trading period the informed stays in the exchange and the uninformed trades in the dark pool). In contrast, we show that when market conditions are such that the informed trader migrates to the dark pool and the uninformed stays in the exchange, the existence of the dark pool harms price informativeness.[6]

Buti et al. (2017) and Brolley (2020) examine the competition between a fully transparent limit order book and a dark pool. In a symmetric information setup with private values, Buti et al. (2017) show that the introduction of a dark pool that competes with an illiquid limit order book is, on average, associated with trade creation, wider spreads, lower depth, and welfare deterioration. To complement their work, we introduce asymmetric information in a common value setup and find the same results as in Buti et al. (2017) for low fundamental volatility stocks (when information is of low value for an informed trader) in the first trading period. However, since traders learn from prices in the second trading period, our market quality results differ fundamentally. In a model with asymmetric information, Brolley (2020) shows that the impact of dark trading on market quality depends on the relative price improvement of dark orders over limit orders. In contrast to Brolley (2020), who studies how different levels of dark pool price improvement affect market quality, we develop a model in which the dark pool reference price is exactly the midpoint of the exchange. We characterize how the effects of a dark pool that competes with an exchange on market quality depend on the market quality indicator, trading period, and stock and trader characteristics. The differences in both trading

[5] Our research is also related to two other broader strands of the literature: competition between multiple trading venues (Pagano, 1989; Chowdry and Nanda, 1991; for a review of the literature, see Gomber et al., 2016) and transparency (Biais, 1993; Madhavan 1995; Frutos and Manzano, 2002, 2005; Dumitrescu, 2010; Boulatov and George, 2013, among others).

[6] Ye (2011) finds that adding a dark pool alongside a dealer market always reduces price informativeness if the uninformed is restricted to trading in the exchange.

periods emerge because in our two-period trading model with long-lived information, an uninformed trader uses the limit order book to extract information about the common value of the asset.

The fact that market performance indicators depend fundamentally on stock and trader characteristics is novel and helps us reconcile the mixed results reported in the empirical literature on the effects of dark pools on the market performance of the exchange. In terms of price informativeness, our results that price informativeness may decrease are consistent with the empirical results reported by Hendershott and Jones (2005), Comerton-Forde and Putniņš (2015) (for high levels of dark trading; i.e., above 10%), Hatheway et al. (2017), and Brogaard and Pan (2019), while our results that price informativeness may increase (in the second trading period) are related to the empirical results reported by Ready (2014) and Comerton-Forde and Putniņš (2015) (for low levels of dark trading). We also show that in the two-venue market in which a low fundamental volatility stock is traded there is a negative effect on liquidity in the first trading period (which is consistent with the empirical studies of Nimalendran and Ray, 2014; Weaver, 2014; Kwan et al., 2015; Degryse et al., 2015; and Hatheway et al., 2017), while when a high fundamental volatility stock is traded there is a positive effect on liquidity (Gresse, 2006; Buti et al., 2011; Ready, 2014). In addition, our model also provides new empirical implications regarding changes in market quality, both in the time-series and the cross-section, emphasizing the role of stock and trader characteristics in the decision on whether to supply or demand liquidity in the exchange.

Our results can also inform policy about the central role played by market conditions in the competition between dark pools and exchanges, and thus, this paper contributes to the ongoing regulatory debate on market structure regulation. This is the case, for example, for the MiFID II rule known as the double volume cap ($DVC$) that limits the volume of dark trading in one venue at 4% and the entire market at 8%. In order to address market participants' concerns about the effectiveness of the new dark trading rules and the increase in trading costs, ESMA (2020) recently decided to transform the double volume cap mechanism by removing the volume cap that limits dark trading in a single venue at 4%, as they concluded that the MiFID II measures in place since January 2018 had failed to curb trading in dark pools.[7]

[7] According to ESMA (2019), the reduction of dark trading volume after the implementation of the double volume cap mechanism was only temporary. Thus, for the equities subject to the ban, the amount of trading executed in dark pools decreased from more than 7% in January 2018 to less than 1% of the total volume in August 2018. However, as the ban ended, investors returned to dark pools and the volume increased to more than 5%. Moreover, instead of going back to the exchange, investors started trading in periodic auctions.

# 1 Model

We consider a market in which a single risky asset is traded. The liquidation value of the asset, $\widetilde{V}$, may take two values, $V \in \{V^H, V^L\}$, with equal probabilities. We denote the unconditional mean of $\widetilde{V}$ by $\mu$ and $\sigma > 0$ represents the fundamental volatility (i.e., standard deviation). The asset may be traded in two venues: an exchange, organized as a limit order book ($LOB$), or a dark pool ($DP$).

At the beginning of the game, potential participants in the exchange have access to an electronic book that provides an anonymous list of previously entered limit orders. Specifically, the initial $LOB$ has at least three prices on the ask and bid sides of the book: $A_1^1$, $A_1^2$, $A_1^3$, and $B_1^1$, $B_1^2$, $B_1^3$, respectively, such that $V^L \leq B_1^3 < B_1^2 < B_1^1 < \mu < A_1^1 < A_1^2 < A_1^3 \leq V^H$. In addition, prices are placed on a grid and the following relationships hold:

$$
\begin{aligned}
A_1^1 &= \mu + k_1\tau,\ A_1^2 = \mu + k_2\tau,\ A_1^3 = \mu + k_3\tau,\ V^H = \mu + \kappa\tau, \\
B_1^1 &= \mu - k_1\tau,\ B_1^2 = \mu - k_2\tau,\ B_1^3 = \mu - k_3\tau,\ V^L = \mu - \kappa\tau,
\end{aligned} \tag{1}
$$

with $1 \leq k_1 < k_2 < k_3 \leq \kappa$, where $k_1$, $k_2$, and $k_3$ are natural numbers, and $\tau$ is the tick size (i.e., the minimum price change that traders are allowed to quote over the existing price). Note that the volatility of the asset satisfies $\sigma = \kappa\tau$, with $\kappa$ being a real number.

Some limit order book models assume that the book starts empty (see, for instance, Seppi, 1997; Buti and Rindi, 2013; Buti et al., 2017; and Riccó et al., 2020), that is, the only standing limit orders in the initial book are those at "extreme" prices coming from a trading crowd. As Riccó et al. (2020) point out this is a simplification given that, in practice, daily opening limit order books include uncancelled orders from the previous day and new limit orders from opening auctions. In these models, in the first trading period, if an investor wants to trade, he always selects a limit order. By contrast, in an initial non-empty $LOB$ (as in Parlour, 1998 and Foucault, 1999) the trader can select any type of order. The way the price grid works in our model allows us to start with a full or an almost empty book depending on the parametrization. Thus, for low values of $k_1$ the book has orders with prices that are close to the midpoint – the mean of the liquidation value of the asset – and therefore, the book is similar to a full book. However, for very high values of $k_1$ close to $\kappa$, traders will behave as if the book was empty.[8] We can interpret $1/k_1$ as a measure of stock liquidity, so the

[8] We do not model a trading crowd willing to provide liquidity at the highest possible prices (as in Seppi, 1997 and Parlour, 1998). In their setup, this assumption prevents traders from bidding prices that are too distant from the inside

market is very liquid when $k_1 = 1$. For simplicity, we assume that the initial depth of the $LOB$ at each bid and ask price is equal to 1, and that the $LOB$ follows price and time priority rules.[9] The $LOB$ is fully transparent (i.e., all of the information in the $LOB$ is available to all market participants at any point in time). Traders can submit market orders or limit orders to the $LOB$. There are no transaction costs or trading fees.

The $DP$ is completely opaque in the sense that an order submitted to the $DP$ is not observable to anyone besides the trader who submitted it. If a trader submits an order to the $DP$ and it is executed at $t$, then the execution price is equal to the midpoint of the exchange at $t$: $(A_t^1 + B_t^1)/2$, where $A_t^1$ and $B_t^1$ denote the best ask and bid prices at the beginning of trading period $t$. If the order is not executed in the $DP$ at $t$, then the trader can cancel it or keep it. If the trader keeps the order, then it returns to the exchange at $t+1$.[10] This feature of our model is novel in the literature and it allows us to model an important trade-off that traders face when submitting a dark order: they obtain a price improvement, but face the risk of non-execution in the dark and the possibility that when their orders return to the market the price has moved against them.

Following Degryse et al. (2009), and to keep the tractability of our model with asymmetric information, we assume that the liquidity supply in the $DP$ at $t = 1$ is exogenous, however, the probability of execution in the dark pool becomes endogenous at $t = 2$. Thus, similarly to the $LOB$, the initial liquidity in the $DP$ is provided by traders outside the model. In practice, the $DP$ order flow is substantially fragmented partly due to the emergence of order routing and splitting algorithms (Gresse, 2017). Thus, $DP$ orders come from multiple and diverse sources, including the order flow broker-dealers, institutional traders, liquidity providers and proprietary flow. Moreover, the orders that traders might send to the $DP$ in our model are small in relation to the aggregate liquidity in the $DP$.[11]

Concerning traders, we assume that all traders are risk neutral and may trade one unit of the asset (as in Glosten and Milgrom, 1985; Foucault, 1999; and Riccó et al., 2020). There are two possible types of traders: rational and liquidity traders.[12] Rational traders choose an order submission strategy that

spread. In our framework, since we assume that there are at least three prices previously populated with orders in the $LOB$ and order size is 1, traders will not get to trade against this crowd.

[9] First, the order with the best price is executed. Second, among the orders with the same price, priority is given to the order that arrives first.

[10] In practice, a trader can implement the decision to cancel or keep an order when it is not executed in the $DP$ using smart order routing. Orders are automatically filled while sweeping for liquidity at the available trading venues.

[11] Aguilar (2015) argues that average trade size on a $DP$ is similar to those traded on exchanges.

[12] Rational traders may be institutional traders that monitor liquidity using algorithms and strategically submit orders (see Malinova et al. 2018), while liquidity traders can be understood as retail investors that typically do not have this

maximizes their expected profits conditional on their information set at each date, $I_t$, which includes information about the liquidation value of the asset and the state of the $LOB$. Rational traders simultaneously select whether or not to trade ($NT$), and if they trade, they choose the trading venue (exchange or $DP$), and the order type in the exchange (market order, $MO$, or limit order, $LO$). $DO$ represents the order type in the $DP$. Consequently, the set of strategies available to a rational trader (both informed and uninformed) is

$$\mathcal{O}_D = \{MO,\ LO,\ DO,\ NT\}, \tag{2}$$

where a $B$ in front of an order type denotes a buy order and a $S$ a sell order.[13] $\Pi^R_{\mathcal{O},t}$ represents the profits of a particular order, where the superscript $R$ denotes that the order comes from a rational trader ($R = I, U$, where $I$ and $U$ indicate informed and uninformed traders, respectively); subscript $\mathcal{O}$ is the order type $\mathcal{O} \in \mathcal{O}_D$ defined in (2), and the subscript $t$ is the order submission date.

The sequence of events is illustrated in Figure 1.

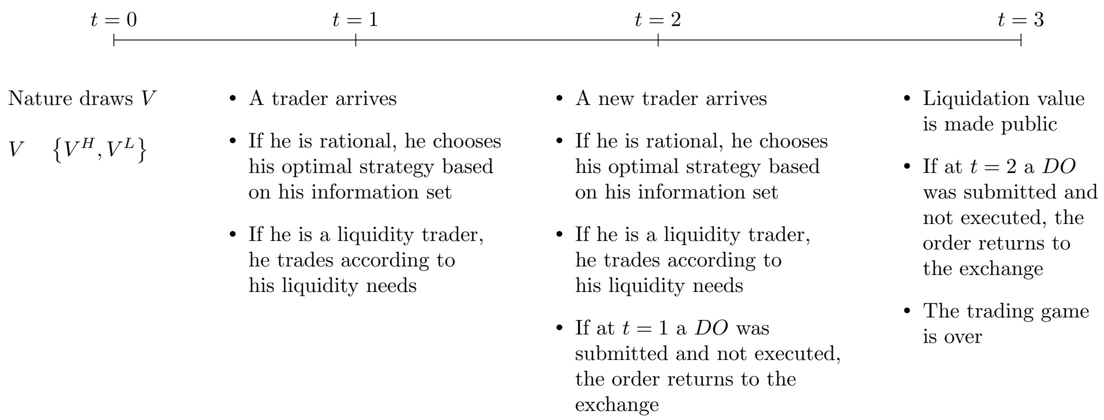


Figure 1: Timeline of the trading game when traders have access to the dark pool.

Figure 2 illustrates the tree of events for the first trading period.[14] A rational trader arrives at the market with probability $\lambda > 0$ and a liquidity trader arrives with probability $1 - \lambda > 0$. Rational traders may be either (privately) informed if they have perfect information about the liquidation value of the asset (with probability $\pi > 0$), or (privately) uninformed if they know only the distribution of the liquidation value of the asset (with probability $1 - \pi$).[15] We use $PIN \equiv \lambda\pi$, the probability

technology and trade due to liquidity needs.

[13]For instance, $BMO$ denotes a buy market order, while $SMO$ a sell market order.

[14]We can draw a similar tree of events for the second trading period.

[15]Uninformed traders are those that only use public information while informed traders have access to both public

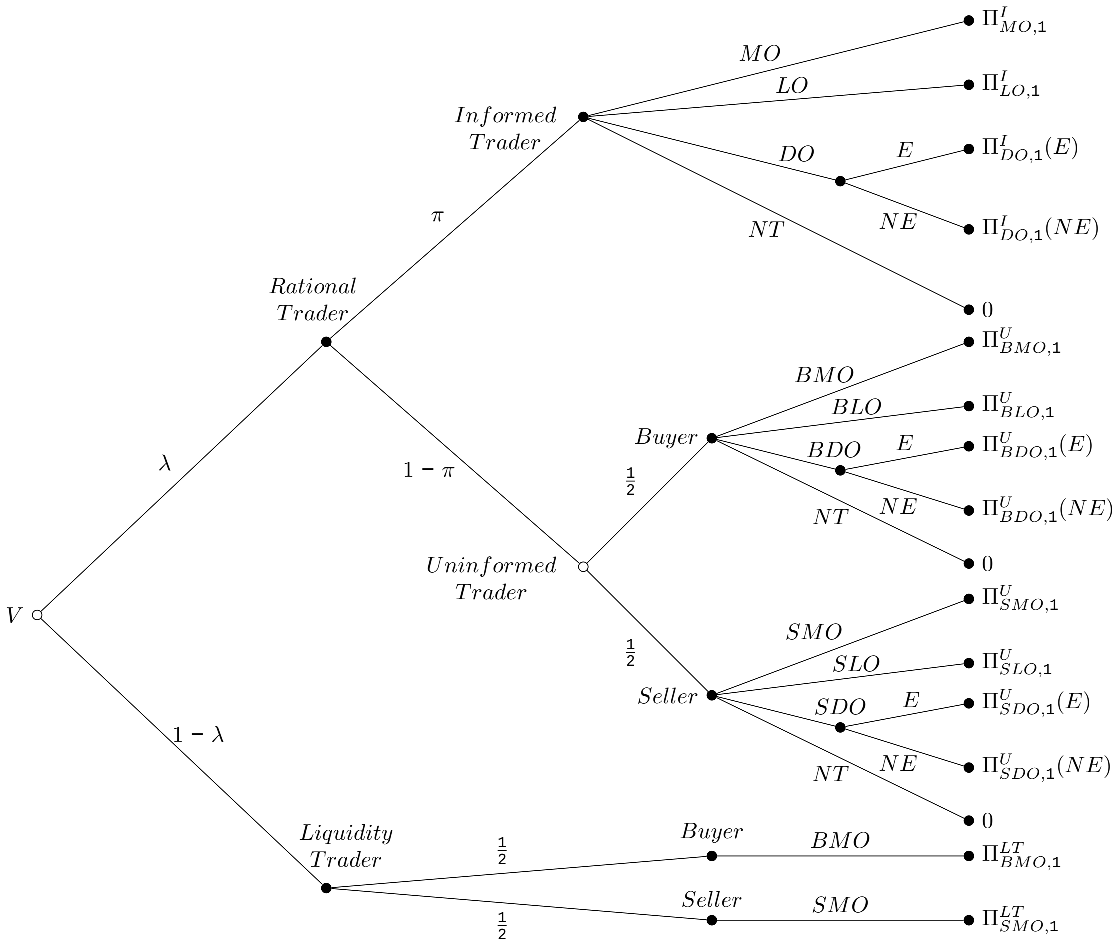


Figure 2: Tree of events of the first trading period.

of informed trading as a measure of information asymmetry, following Easley and O'Hara (1987) and Easley et al. (1996). An informed trader buys when observing $V = V^H$ (denoted by $IH$), and sells when observing $V = V^L$ (denoted by $IL$). An uninformed trader is a buyer (denoted by $UB$) with probability $\frac{1}{2}$ or a seller (denoted by $US$) with probability $\frac{1}{2}$. A liquidity trader buys with probability $\frac{1}{2}$ or sells with probability $\frac{1}{2}$ for liquidity or hedging needs. Notice that uniformed traders are rational and this implies that they can choose the type of order and venue. Moreover, the uninformed can learn from the changes in the $LOB$, so their orders may change from one period to another. By contrast, liquidity traders are not rational, and they are modelled to foster the trading of uninformed traders.

and private information. For example, an uninformed trader could be a fund manager that rebalances his portfolio for non-informational reasons (see Han et al., 2016), while an informed trader may be a fund manager who uses his connections to acquire information (see Coval and Moskowitz, 2001; Cohen et al. 2008 among others).

The final nodes of the tree include the profits for each of the trading options at $t = 1$. The structure of the model and distributions of random variables are common knowledge.

For each possible order type, we next examine its characteristics and the associated expected profits for a rational buyer (the sell order profits are analogous). Internet Appendix I derives in detail the expected profits of all traders at all times and for all possible states of the $LOB$.

- Market order ($MO$): It is executed immediately at the given best available ask/bid prices. The expected profits of a $BMO$ at date $t$ are

$$\mathsf{E}\left(\Pi^R_{BMO,t}|I_t\right) = \mathsf{E}\left(\widetilde{V}|I_t\right) - A^1_t. \tag{3}$$

- Limit order ($LO$): A $LO$ that improves the current market price may be executed in the next period if a $MO$ of the opposite sign hits the $LOB$. Thus, $LO$s provide better prices than $MO$s do, but have execution risk. When a trader chooses a $LO$, this order always improves the current price by one tick because: (i) it is never optimal for the trader to improve the price by more than one tick since it reduces his profits; (ii) it is never optimal for the trader to submit a non-improving $LO$ since the order is not executed (due to time priority, the order goes to the end of the queue), and obtains zero profits. Given that we also assume a discount factor, $\delta \in (0, 1]$, which is common across traders and periods, the expected profits of a $BLO$ at date $t$ are

$$\mathsf{E}\left(\Pi^R_{BLO,t}|I_t\right) = \delta p^R_{BLO,t}\left(I_t\right)\left(\mathsf{E}\left(\widetilde{V}|I_t\right) - B^1_t - \tau\right), \tag{4}$$

  where $p^R_{BLO,t}$ is the probability of execution of a $BLO$ submitted by a rational trader, $R$, at time $t$.

- Dark order ($DO$): With probability $\theta^R_t$, an order submitted by a rational trader, $R$, to the $DP$ at time $t$ is executed, and with probability $(1 - \theta^R_t)$ it is not executed. Since no new trader arrives in the market at $t = 3$ an order that returns to the exchange from the $DP$ at the end $t = 2$ will be either a $MO$ (we call this dark order $BDO - MO$) or $NT$ (we call this order $BDO - NT$).[16] We denominate the dark order $DO$ as the best of the two dark orders $BDO - MO$ and $BDO - NT$

[16]This is because the probability of execution of a $LO$ at $t = 3$ is $0$. Hence, an order will never return to the market as a $LO$.

(for each type of trader).[17] Note that a $DO$ does not change the state of the $LOB$, and to model the reporting delay of $DP$ trades, we consider that the $DP$ does not report trades until the end of the trading game. Therefore, the expected profits of a $BDO$ submitted at time $t$ are

$$\begin{aligned} \mathsf{E}\left(\Pi^R_{BDO,t}|I_t\right) &= \max\left\{\mathsf{E}\left(\Pi^R_{BDO-MO,t}|I_t\right), \mathsf{E}\left(\Pi^R_{BDO-NT,t}|I_t\right)\right\} \\ &= \theta^R_t\left(\mathsf{E}\left(\widetilde{V}|I_t\right) - \frac{A^1_t + B^1_t}{2}\right) + (1-\theta^R_t)\delta\max\left\{\mathsf{E}\left(\Pi^R_{BMO,t+1}|I_t\right), 0\right\}. \end{aligned} \tag{5}$$

- No trade ($NT$): A trader who refrains from trading at $t$ obtains zero profits:

$$\mathsf{E}\left(\Pi^R_{NT,t}|I_t\right) = 0. \tag{6}$$

In case of equal profits, we assume that a $MO$ dominates both a $LO$ and a $DO$, and a $LO$ dominates a $DO$. If the expected profits of a $MO$ are null, then a rational trader refrains from trading.

We can represent our model by a two-period game of incomplete information, and we therefore use the Perfect Bayesian Equilibrium ($PBE$) concept. In the following, we focus on a symmetric $PBE$ in pure strategies, hereafter, equilibrium. A symmetric equilibrium refers to a situation in which buyers and sellers with the same information (i.e., informed or uninformed) choose the same order type (except the direction of trade).

## 2 Equilibrium in the single-venue market model

We first consider the single-venue market - where traders can only trade in the exchange. Hence, the set of strategies available to a rational trader is $\mathcal{O}_D \setminus \{DO\}$, that is, a $MO$, a $LO$, and $NT$.

We solve the game backwards. Since the buy and sell sides are separable and symmetric in this model, we focus for exposition on the buy side. The expected profits for the rational traders at $t = 2$ are summarized in Appendix B, Tables B.1 and B.2, while Tables B.4 and B.5 display the expected profits for these traders at $t = 1$. The following lemma presents the informed and uninformed traders' optimal choices at $t = 2$ and $t = 1$.

[17] As we show in the Internet Appendix I, when an informed trader chooses a $DO$ at $t = 1$ and the order is not executed, it is optimal for the informed trader to choose a $MO$ when the order returns to the exchange at the end of the second trading period (i.e., $DO - MO$). In contrast, when an uninformed trader chooses a $DO$ at $t = 1$ and the order is not executed, it is optimal for the uninformed to cancel it at the end of the second trading period (i.e., $DO - NT$). However, at $t = 2$ both types of traders are indifferent between $DO - MO$ and $DO - NT$ since at $t = 3$ the liquidation value is revealed and the profits of both strategies are zero.

**Lemma 1** *In equilibrium, the following results hold:*

- *at $t = 2$: An informed trader always submits a $MO$, while an uninformed trader may submit either a $MO$ or $NT$, but never chooses a $LO$.*
- *at $t = 1$: An informed trader may submit either a $MO$ or a $LO$, but never chooses $NT$, while an uniformed trader may submit either a $LO$ or $NT$, but never chooses a $MO$.*

An informed trader at $t = 2$ always chooses a $MO$ since it generates positive expected profits, while the expected profits of a $LO$ or $NT$ are always null. In this trading period, an uninformed trader never chooses a $LO$ since the probability of execution of this type of order is null given that no new orders arrive thereafter. Consequently, the uninformed trader's choice at $t = 2$ will be either a $MO$ or $NT$, depending on the information gathered from the state of the $LOB$. When the state of the $LOB$ conveys no information, then the optimal choice is $NT$ since the expected profits of a $MO$ are negative. If the $LOB$ reveals that a $BMO$ or a $BLO$ was submitted at $t = 1$, then the uninformed buyer at $t = 2$ chooses a $BMO$ if his belief that the order at $t = 1$ came from an informed trader is sufficiently strong, which results in positive expected profits from submitting a $BMO$. In addition, if the state of the $LOB$ reveals that the trader at $t = 1$ submitted a $SMO$ or $SLO$, then the uninformed trader's expected profits from submitting a $BMO$ at $t = 2$ are negative, and hence, the trader chooses $NT$.

At $t = 1$, an informed trader never chooses $NT$ since it is always dominated at least by a $MO$, and hence, an informed trader may choose either a $MO$ or $LO$. In contrast, an uninformed trader at $t = 1$ never selects a $MO$ since the expected profits are negative, and hence, it is always dominated at least by $NT$. Consequently, an uninformed trader at $t = 1$ may choose either a $LO$ or $NT$.

Thus, the candidate strategy profiles at $t = 1$ that can be sustained as a symmetric $PBE$ are:

$$\mathsf{E}_1^{ND} : (BMO,\ SMO,\ BLO,\ SLO), \qquad \mathsf{E}_2^{ND} : (BMO,\ SMO,\ NT,\ NT),$$

$$\mathsf{E}_3^{ND} : (BLO,\ SLO,\ BLO,\ SLO), \qquad \mathsf{E}_4^{ND} : (BLO,\ SLO,\ NT,\ NT),$$

where the first two components correspond to the strategies of informed traders at $t = 1$ ($IH$ and $IL$, respectively) and the last two components correspond to the strategies of uninformed traders at $t = 1$ ($UB$ and $US$, respectively).[18]

[18] In what it follows the superscript $ND$ indicates that there is no access to the dark pool, while $D$ indicates that there is access.

The next proposition characterizes the symmetric $PBE$ of the reduced trading game in the single-venue market.

**Proposition 1** *In the single-venue market: Case A. If $k_1 > 1$, then the optimal strategy profiles at $t = 1$ are:*

$$\begin{cases} (BLO, SLO, BLO, SLO) & \text{if} \quad \sigma < \kappa^I_{MO \quad LO}\tau \text{ and } PIN < \psi^U_{LO \quad NT}, \\ (BLO, SLO, NT, NT) & \text{if} \quad \sigma < \kappa^I_{MO \quad LO}\tau \text{ and } PIN \quad \psi^U_{LO \quad NT}, \\ (BMO, SMO, BLO, SLO) & \text{if} \quad \kappa^I_{MO \quad LO}\tau \quad \sigma \text{ and } PIN < \psi^U_{LO \quad NT}, \\ (BMO, SMO, NT, NT) & \text{if} \quad \kappa^I_{MO \quad LO}\tau \quad \sigma \text{ and } PIN \quad \psi^U_{LO \quad NT}, \end{cases}$$

*where* $\kappa^I_{MO \quad LO} \quad (k_1 \quad 1) + 2\frac{\delta(k_1 \quad 1)(1 \quad \lambda)+1}{2 \quad \delta(1 \quad \lambda)}$, $PIN \quad \lambda\pi$, *and* $\psi^U_{LO \quad NT} \quad \frac{(1 \quad \lambda)(k_1 \quad 1)\tau}{\sigma \quad (k_1 \quad 1)\tau}$.

*Case B. If $k_1 = 1$ (the asset is very liquid), then the optimal strategy profile at $t = 1$ is $(BMO, SMO, NT, NT)$.*

*For Cases A and B, the optimal strategy of an informed trader at $t = 2$ is to choose a $MO$ for all possible states of the $LOB$. Table B.3 in Appendix B describes the optimal strategy of an uninformed trader at $t = 2$. The equilibrium beliefs of an uninformed trader at $t = 2$ are given in the proof of this proposition.*

**Remark 1** *Notice that $\kappa^I_{MO \quad LO}$ denotes the minimum value of $\kappa$ such that at $t = 1$ an informed trader chooses a $MO$ instead of a $LO$, while $\psi^U_{LO \quad NT}$ represents the minimum value of $PIN$ such that at $t = 1$, an uninformed trader chooses $NT$ instead of a $LO$.*

In the second trading period, according to Lemma 1, Proposition 1 shows that an informed trader submits a $MO$ for all states of the $LOB$. An uninformed trader chooses $NT$, except if the state of the $LOB$ conveys information about the fundamental value of the asset and he strongly believes that a $MO$ or $LO$ of the same direction as the order was submitted by an informed trader at $t = 1$. In this case, the uninformed trader submits a $MO$.

In the first trading period, Proposition 1 indicates that when the fundamental asset volatility is sufficiently low (i.e., $\sigma < \kappa^I_{MO \quad LO}\tau$), it is optimal for an informed trader to supply liquidity (i.e., to place a $LO$), while the decision of the uniformed trader depends on the severity of the adverse selection problem. Therefore, there are two possible optimal strategy profiles when the asset has low volatility: $(BLO, SLO, BLO, SLO)$ and $(BLO, SLO, NT, NT)$. The optimal strategy profile $(BLO, SLO, BLO, SLO)$ occurs in a market with low adverse selection risk (either because the asset's volatility is extremely low, or both the asset's volatility and the $PIN$ are low at the same time). When the adverse selection problem is sufficiently high (because the asset's volatility is not low, and the $PIN$ is high enough) the optimal strategy profile is $(BLO, SLO, NT, NT)$. In particular, when the probability of informed trading is low, uninformed traders realize that by placing a $LO$ at the exchange in the first trading period, they are very unlikely to end up trading with informed traders. Combining the fundamental volatility and information asymmetry dimensions, we call these stocks *"Low-Low"* (i.e., low fundamental volatility– low $PIN$), and *"Low-High"*, respectively. In the subsequent analysis, we sometimes consider only one of these dimensions in isolation, such as low/high fundamental volatility stocks or low/high $PIN$.

By contrast, when the fundamental asset volatility is sufficiently high (i.e., $\sigma \quad \kappa^{I}_{MO \quad LO}\tau$), it is optimal for the informed trader to demand liquidity (i.e., to place a $MO$) in the first trading period. Note that the informational advantage of an informed trader increases with the volatility of the asset ($\sigma$). Thus, when the asset's volatility is sufficiently high, an informed trader prefers immediate execution ($MO$). When $\sigma$ is not high enough, the informed trader selects a $LO$ because of its price improvement. Furthermore, the uniformed trader's decision depends again on the level of information asymmetry. Consequently, in the case of high volatility, there are two possible optimal strategies: $(BMO, SMO, BLO, SLO)$ and $(BMO, SMO, NT, NT)$. The strategy $(BMO, SMO, BLO, SLO)$ is optimal when the degree of information asymmetry is sufficiently low (i.e., $PIN < \psi^{U}_{LO \quad NT}$), while $(BMO, SMO, NT, NT)$ is optimal when the degree of information asymmetry is sufficiently high (i.e., $PIN \quad \psi^{U}_{LO \quad NT}$). We call these stocks *"High-Low"*, and *"High-High"*, respectively.

Figure 3 illustrates the optimal strategies in the first trading period of the single-venue market.[19] Numerical simulations show that: (i) the higher the asset's volatility is, the lower the probability of informed trading needs to be for an uninformed trader to choose $NT$. (ii) the strategy profile $(BLO, SLO, NT, NT)$ is possible only for very specific parameter configurations, such as for $\pi > 0.5$. For this purpose, in the Internet Appendix IV, we show in Figure IV.2 the optimal strategies at $t = 1$ for parameter values that display the four possible equilibria.

The results derived in Proposition 1 are consistent with the previous work by Goettler et al. (2009),

[19]Figure IV.1 in the Internet Appendix IV shows a similar figure for a liquid market.

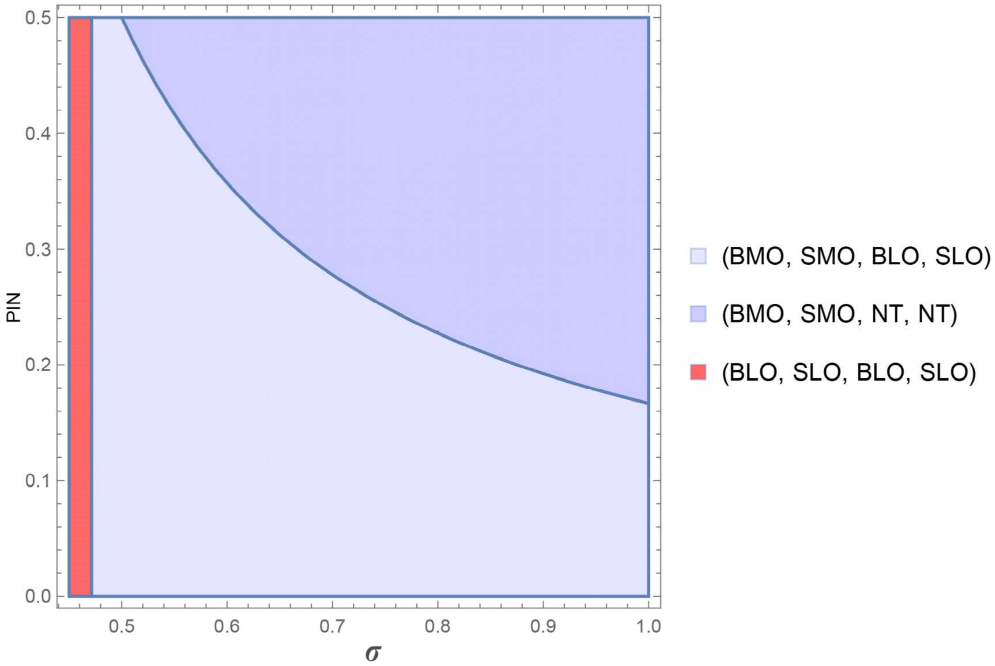


Figure 3: Optimal strategies at $t = 1$ in the single-venue market. Parameters values: $k_1 = 6$, $\lambda = 0.5$, $\tau = 0.05$, $\delta = 0.95$.

who show that informed traders switch from supplying to demanding liquidity when volatility changes from low to high.[20] Interestingly, our model encompasses both the model of Zhu (2014) and Buti et al. (2017). Note that when fundamental volatility and the $PIN$ are high, that is, a *"High-High"* stock, the optimal strategy for an informed trader at $t = 1$ is to place a $MO$ as in Zhu (2014). Similarly, when the probability of having an informed trader is very small ($\pi\ !\ 0$), the model is similar to that in Buti et al. (2017), in which there is no asymmetric information. Notice also that when the asset's volatility is low and $\pi\ !\ 0$, traders choose $LO$ at $t = 1$, so the prevailing equilibrium is similar to $E_3^{ND}$. Note that a *"High-Low"* stock corresponds to equilibrium $E_1^{ND}$; a *"High-High"* to $E_2^{ND}$; a *"Low-Low"* to $E_3^{ND}$; and a *"Low-High"* to $E_4^{ND}$.

The following corollary describes the comparative statics of $\kappa^I_{MO\ \ LO}$ and $\psi^U_{LO\ \ NT}$ with respect to various market and trader characteristics.

**Corollary 1** *Ceteris paribus, $\kappa^I_{MO\ \ LO}$ increases with $\delta$ and $k_1$, and decreases with $\lambda$, while $\psi^U_{LO\ \ NT}$ increases with $k_1$, and decreases with $\lambda$ and $\kappa$.*

The corollary above implies that for the informed trader at $t = 1$, an increase in the discount

[20] Goettler et al. (2009) point out that first as volatility increases, the risk of a $LO$ increases, as they are more likely to be picked-up for trading. Second, as volatility increases, so does the likelihood of finding mispriced orders in the $LOB$.

factor, a decrease in the liquidity of the asset ($1/k_1$), or an increase in the probability that a liquidity trader arrives at $t = 2$ (ceteris paribus) reduces the relative attractiveness of a $MO$ compared to a $LO$ for an informed trader.[21] Regarding the uninformed trader at $t = 1$, a decrease in the liquidity of the asset, an increase in the probability that a liquidity trader arrives at $t = 2$, or a reduction in the volatility of the asset at $t = 2$ (ceteris paribus) increases the attractiveness of a $LO$ with respect to $NT$.

Note that according to Corollary 1, the condition $\sigma < \kappa^I_{MO \quad LO}\tau$ can be satisfied, ceteris paribus, for a low fundamental volatility or low liquidity stock (high $k_1$), or when rational traders are characterized by low immediacy (high $\delta$) or participate as a relatively small proportion of the market (small $\lambda$). In addition, note that our classification of high/low fundamental volatility stocks also depends on the tick size: ceteris paribus, as the tick size increases, the low fundamental volatility region expands. To sum up, the characterization of stocks as *"High"* and "*Low"* in terms of liquidity, immediacy, or the proportion of rational traders gives analogous results to the characterization in terms of *"High"* and "*Low"* fundamental volatility. For simplicity, we illustrate our results by discussing them in terms of the fundamental asset volatility, but a similar analysis is possible by studying changes in other stock market and trader characteristics.

## 3 Equilibrium in the two-venue market model

We next consider a two-venue market model in which rational traders have access to both the exchange and the $DP$. Hence, the orders they can submit are given in (2).

The decision to submit an order to the $DP$ depends on its probability of execution in the dark pool. We denote $\theta^I_t$ and $\theta^U_t$ as the probability of execution of a $DO$ at trading period $t$ for an informed and uninformed trader, respectively. In our model, the probability of execution in the $DP$ is exogenous in the first trading period and it depends on the order imbalance in the dark pool. However, in the second trading period, this probability is endogenous since it depends on the order imbalance as well as on the traders' actions at $t = 1$. To illustrate this, we consider the case in which the prior probability distribution of the order imbalance in the $DP$ is common for all rational traders, and our result is

[21] The informed trader's profits at $t = 1$ do not depend on the probability that an informed trader arrives in the next trading period, $\lambda\pi$. This is because an informed trader that submits a $LO$ at $t = 1$ knows that the $LO$ will not be executed in the next trading period against an order submitted by an informed trader, since an informed trader at $t = 2$ chooses an order of the same sign as the initial order. In addition, an informed trader at $t = 1$ correctly predicts that an uninformed trader at $t = 2$ never submits a $MO$ of the opposite sign as the informed trader at $t = 1$.

below.

**Lemma 2** *Suppose that the prior probability distribution of the order imbalance in the DP is common for all rational traders. Then, the probability of execution of a DP order in the second trading period is greater or equal for an uninformed than for an informed trader; that is,* $\theta_2^I \quad \theta_2^U$.

If in the first trading period a trader chooses a $DO$, then the informed trader at $t = 2$ is aware that the direction of the order submitted by an informed trader at $t = 1$ is the same as his order. However, for an uninformed trader, the coincidence in order direction between periods does not have to be fulfilled. Consequently, it is less likely that a $DO$ of an informed trader is executed in the $DP$ at $t = 2$ compared to that of an uninformed trader ($\theta_2^I < \theta_2^U$). This difference ($\theta_2^U \quad \theta_2^I$) increases with the $PIN$. In addition, note that when no trader chooses a $DO$ at $t = 1$, then the probability of execution in the $DP$ at $t = 2$ is the same for informed and uninformed traders ($\theta_2^I = \theta_2^U$). This example is similar to the mechanism in Zhu (2014), although in our setup, the informed trader goes to the $DP$ under some market conditions.

As in the previous section, we solve the model backwards. First, we calculate the expected profits for the informed and uniformed traders at $t = 2$ and $t = 1$, and summarize the results in Appendix C, Tables C.1, C.2, C.3, and C.4, respectively. Comparing the expected profits of each of the possible orders for each type of rational trader at $t = 2$ and $t = 1$, Lemma 3 states the strategies that are dominated, and hence, never chosen by a rational player.

**Lemma 3** *In equilibrium, the following results hold:*

- *At* $t = 2$*: An informed trader may submit either a* $MO$ *or a* $DO$*, but never chooses a* $LO$ *or* $NT$*, while an uninformed trader may submit either a* $MO$*, a* $DO$*, or* $NT$*, but never chooses a* $LO$*.*

- *At* $t = 1$*: An informed trader may submit either a* $MO$*, a* $LO$ *or a* $DO$*, but never chooses* $NT$*, while an uninformed trader may submit either a* $LO$ *or* $NT$*, but never chooses a* $MO$ *or a* $DO$*.*

We find that in the second trading period, a $LO$ is never chosen since it is never executed: a) if the $LOB$ changes, then no $MO$ arrives at the end of the second trading period, and hence, a $LO$ has zero probability of execution; b) if the $LOB$ did not change, then a $LO$ can only be executed if an uninformed trader at $t = 1$ chooses a $DO$, but this cannot occur in equilibrium since the expected profits are null.[22] Moreover, an informed trader at $t = 2$ never chooses $NT$ since it is always dominated by a $MO$.

In the first trading period, an informed trader never chooses $NT$ since it is always dominated by at least a $MO$. Moreover, the expected profits of a $DO$ submitted by an informed trader at $t = 1$ are strictly positive (see Table C.3 in the Appendix C), and hence, a $DO$ might be optimal for the informed trader at $t = 1$. By contrast, an uninformed trader at $t = 1$ may choose between a $LO$ or $NT$ since the expected profits of a $MO$ are negative and those of a $DO$ are null (see Table C.4 in the Appendix C).[23]

Hence, the sustainable candidate strategy profiles at $t = 1$ as a $PBE$ are:

$$E_1^D : (BMO, SMO, BLO, SLO), \quad E_2^D : (BMO, SMO, NT, NT),$$

$$E_3^D : (BLO, SLO, BLO, SLO), \quad E_4^D : (BLO, SLO, NT, NT),$$

$$E_5^D : (BDO, SDO, BLO, SLO), \quad E_6^D : (BDO, SDO, NT, NT),$$

where, as before, the first two components correspond to the strategies of informed traders at $t = 1$ ($IH$ and $IL$, respectively) and the last two components correspond to strategies of uninformed traders at $t = 1$ ($UB$ and $US$, respectively).

Next, we characterize the equilibrium of the trading game in the two-venue market.

**Proposition 2** *In the two-venue market:*

*Case A. Suppose $k_1 > 1$. Then, we have the following cases:*

*Case A.1 If $\sigma < \kappa^I_{MO \quad LO}\tau$ and $PIN < \psi^U_{LO \quad NT}$, then the optimal strategy profiles at $t = 1$ are:*

$$\begin{cases} (BLO, SLO, BLO, SLO) & \text{if } \theta^I_1 \text{ is sufficiently small,} \\ (BDO, SDO, BLO, SLO) & \text{if } \theta^I_1 \text{ is sufficiently large.} \end{cases}$$

[22]The uninformed trader at $t = 2$ forms the correct beliefs that if a $LO$ is executed at the end of this trading period, then his counterparty must be the informed trader who arrived at $t = 1$ with probability 1. However, this information reveals to the uninformed buyer (seller) that the value of the asset must be low (high), and hence, the expected profits of a $LO$ are negative.

[23]Note that the mechanism is similar to those in Menkveld et al. (2017) and Brolley (2020): investors weigh each order's execution risk against the price impact. However, in our model, price impact or execution risk are endogenously determined by optimal trading strategies at $t = 1$ and $t = 2$ as traders learn from the $LOB$.

*Case A.2* *If* $\sigma < \kappa^I_{MO\quad LO}\tau$ *and* $PIN \quad \psi^U_{LO\quad NT}$*, then the optimal strategy profiles at* $t = 1$ *are:*

$$\begin{cases} (BLO, SLO, NT, NT) & \text{if} \quad \theta^I_1 \text{ is sufficiently small,} \\ (BDO, SDO, NT, NT) & \text{if} \quad \theta^I_1 \text{ is intermediate,} \\ (BDO, SDO, BLO, SLO) & \text{if} \quad \theta^I_1 \text{ is sufficiently large.} \end{cases}$$

*Case A.3* *If* $\kappa^I_{MO\quad LO}\tau \quad \sigma$ *and*$PIN < \psi^U_{LO\quad NT}$*, then the optimal strategy profiles at* $t = 1$ *are:*

$$\begin{cases} (BMO, SMO, BLO, SLO) & \text{if} \quad \theta^I_1 \text{ is sufficiently small,} \\ (BDO, SDO, BLO, SLO) & \text{if} \quad \theta^I_1 \text{ is sufficiently large.} \end{cases}$$

*Case A.4* *If* $\kappa^I_{MO\quad LO}\tau \quad \sigma$ *and* $PIN \quad \psi^U_{LO\quad NT}$*, then the optimal strategies profile at* $t = 1$ *are:*

$$\begin{cases} (BMO, SMO, NT, NT) & \text{if} \quad \theta^I_1 \text{ is sufficiently small,} \\ (BDO, SDO, NT, NT) & \text{if} \quad \theta^I_1 \text{ is intermediate,} \\ (BDO, SDO, BLO, SLO) & \text{if} \quad \theta^I_1 \text{ is sufficiently large.} \end{cases}$$

*Case B.* *Otherwise, if* $k_1 = 1$ *(the asset is very liquid), then the optimal strategy profiles at* $t = 1$ *are:*

$$\begin{cases} (BMO, SMO, NT, NT) & \text{if} \quad \theta^I_1 \text{ is sufficiently small,} \\ (BDO, SDO, NT, NT) & \text{if} \quad \theta^I_1 \text{ is sufficiently large.} \end{cases}$$

*The proof in Appendix C characterizes the threshold values of* $\theta^I_1$ *for which each strategy profile is optimal at* $t = 1$.

*For Cases A and B, the informed and uninformed traders' optimal choices at* $t = 2$*, and the beliefs of the uninformed traders at* $t = 2$ *are characterized in Lemma C.1 in Appendix C.*

We start backwards by discussing the second trading period. An informed trader submits a $MO$ (a $DO$) for all states of the $LOB$ when the execution risk in the $DP$ is sufficiently high (low) in relation to the price improvement. As the execution risk in the $DP$ decreases, an informed buyer replaces a $BMO$ with a $BDO$ in the following order according to the state of the $LOB$: $(A^2_1, B^1_1), (A^1_1, B^2_1), (A^1_1, B^1_1), (A^1_1, B^1_1 + \tau), (A^1_1 \quad \tau, B^1_1)$. This occurs because when a $BMO$ was submitted at $t = 1$, the gain from another $BMO$ is small in relation to a $BDO$ even though the execution risk in the $DP$ is relatively high. However, when a $SLO$ was submitted at $t = 1$, the gain from a

$BMO$ is large in relation to that from a $BDO$ despite the fact that the execution risk in the $DP$ is relatively low.

For an uninformed trader at $t = 2$, the optimal strategy depends critically on his beliefs about the probability that a $MO$, a $LO$ or a $DO$ order was submitted by an informed trader at $t = 1$. When the state of the $LOB$ contains no information; that is, $(A_1^1, B_1^1)$, then an uninformed trader at $t = 2$ chooses $NT$ since the expected profits of a $MO$ are negative, and the expected profits of a $DO$ are zero because the midpoint price is equal to the unconditional expected liquidation value of the asset. However, an uninformed trader may also choose a $MO$ or a $DO$ in the second trading period if the $LOB$ conveys good news to the trader about the fundamental value of the asset. Note that, in contrast to the first trading period, if the probability of execution in the $DP$ at $t = 2$ is sufficiently high, then an uninformed trader may migrate to the $DP$.

In the first trading period, Proposition 2 shows that having access to a $DP$ may change the optimal submission strategy profiles for informed and uninformed traders. When the probability of execution in the $DP$ for informed traders at $t = 1$ (i.e., $\theta_1^I$) is sufficiently high, an informed trader switches trading venue, from the exchange to the $DP$. Otherwise, if the corresponding probability of execution in the $DP$ is sufficiently low, then an informed trader submits the same types of orders to the exchange ($MO$ or $LO$) as in the single-venue market. The threshold values of the probability of execution in the $DP$ reflect the price improvement and execution trade-off of each order type. In case of execution, the best price is achieved by a $LO$, followed by a $DO$, and the worst price is given by a $MO$. While a $LO$ has execution risk, the $MO$ and $DO$ do not face execution risk for an informed trader. Note that at $t = 1$, a $DO$ faces no risk of execution since we find that if the $DO$ is not executed in the first trading period, then the informed trader routes it back to the exchange as a $MO$ at the end of the second trading period. However, when this order returns to the exchange, it faces the risk that the price worsened because of the order submitted by the trader that arrives at $t = 2$.

Although an uninformed trader never goes to the $DP$ in the first trading period, as presented in Lemma 3, Proposition 2 shows that the existence of the $DP$ might change the optimal strategy of an uninformed trader when the probability of execution in the $DP$ for an informed trader is high enough, since an uninformed trader may switch from $NT$ to a $LO$. This is because the low execution risk in the $DP$ encourages an informed trader at $t = 2$ to trade in the $DP$ rather than in the exchange. Consequently, the adverse selection the uninformed trader faces in the exchange decreases at $t = 1$,

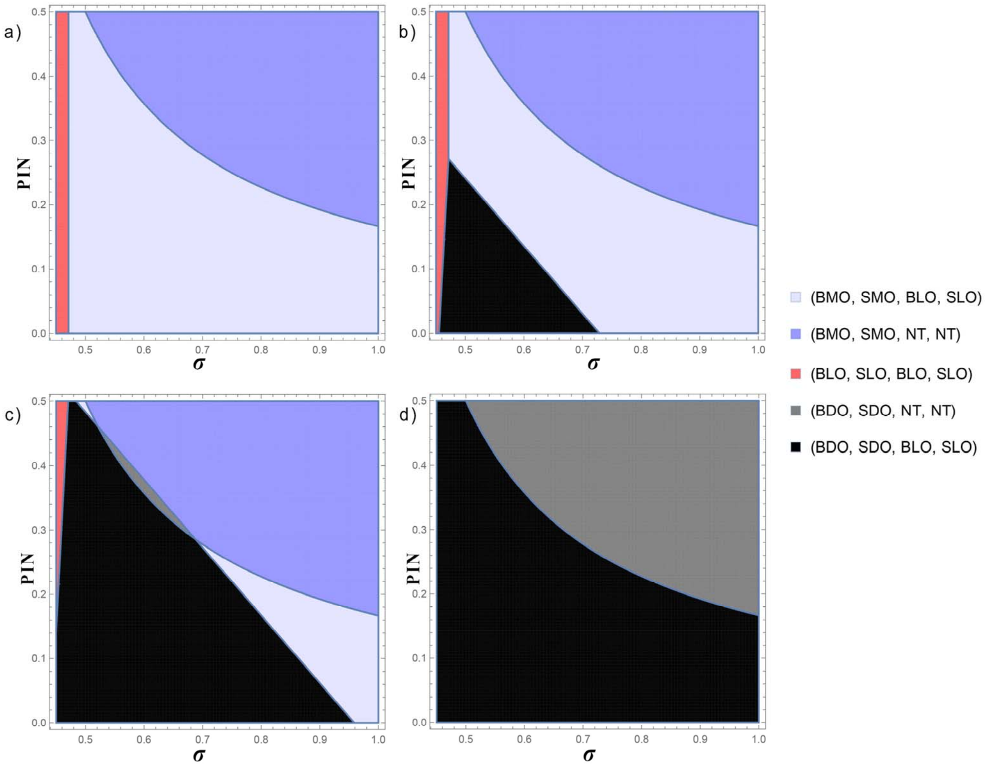


Figure 4: Optimal strategies at $t = 1$ with dark pool. Parameters values: $k_1 = 6$, $k_2 = 7$, $\lambda = 0.5$, $\tau = 0.05$, and $\delta = 0.95$. In Panel a) $\theta_1^I = 0.05$, in Panel b) $\theta_1^I = 0.10$, in Panel c) $\theta_1^I = 0.13$, and in Panel d) $\theta_1^I = 0.25$.

which makes him to trade.

Proposition 2 suggests that restricting the informed trader to participate in the $DP$ might harm the uninformed trader. To illustrate this point, notice that Cases A.2 and A.4 of this proposition show that a significant reduction of $\theta_1^I$ might discourage the uninformed trader from participating in the exchange in the first trading period.

Figure 4 illustrates the optimal strategies at $t = 1$ with respect to the fundamental asset's volatility and information asymmetry for several values of $\theta_1^I$ shown in Panels a), b), c), and d), respectively.[24] In Panel a), the graph has the same features as in Figure 3 since for small values of $\theta_1^I$ there is no migration to the $DP$. In Panel b) and c) we notice that there is a region of parameters in which orders migrate to the $DP$. Thus, $(BLO, SLO, BLO, SLO)$, $(BMO, SMO, LO, LO)$ or $(BMO, SMO, NT, NT)$ prevail when $PIN$ is high and the execution probability in the dark is relatively low. As this probability increases, there is migration to the dark - either ($BDO$, $SDO$, $BLO$, $SLO$) or ($BDO$, $SDO$, $NT$, $NT$) prevail, depending on the initial conditions. Notice that as the fundamental volatility increases,

[24]Figure IV.3 in the Internet Appendix IV shows similar figures for a liquid market.

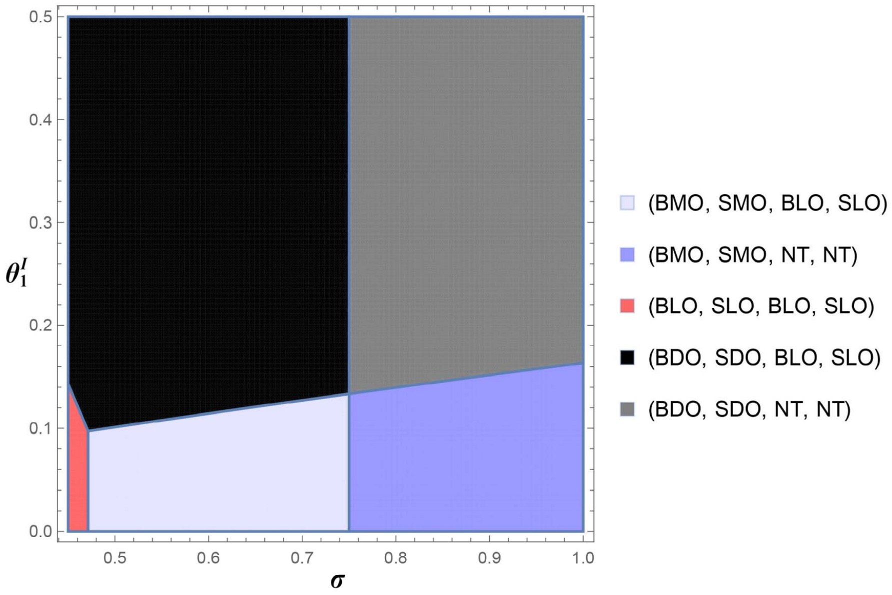


Figure 5: Optimal strategies at $t = 1$ with dark pool. Parameters values: $k_1 = 6$, $k_2 = 7$, $\lambda = 0.5$, $\pi = 0.5$, $\tau = 0.05$, and $\delta = 0.95$.

the informational advantage of the informed trader becomes higher, and hence, this trader has more incentives to trade immediately. This is because the price improvement of a $DO$ does not compensate the risk of not being executed in the dark pool at $t = 1$ and returning to the $LOB$, where the price might have worsened. In Panel d), we find that the informed trader fully migrates to the $DP$ at $t = 1$, while the uninformed trader decides not to trade whenever the adverse selection he faces is high enough.

Figure 5 illustrates the optimal strategy profiles at $t = 1$ with respect to the asset's volatility and the probability of execution for the informed trader in the $DP$ in the first trading period ($\theta_1^I$) for selected parameter values.[25] When $\theta_1^I$ is small, the optimal strategy profiles at $t = 1$ coincide with those in the single-venue market. When an informed trader chooses a $MO$ at $t = 1$, the graph shows that the threshold value of $\theta_1^I$ that leads to the migration of an informed trader's $MO$ to a $DO$ increases with the fundamental asset volatility. In contrast, when the informed trader chooses a $LO$ at $t = 1$, the threshold value of $\theta_1^I$ that leads a switch from a $LO$ to a $DO$ decreases with the volatility.[26]

[25] In addition to the parameter values defined in the captions of Figures 4 and 5, we assume that the beliefs at $t = 2$ are such that an uninformed buyer (seller) does not select a $BLO$ ($SLO$) when there is no change in the $LOB$ prices, and that an informed buyer (seller) chooses a $BMO$ ($SMO$) at $t = 2$ when the $LOB$ has not changed. Furthermore, we assume that the probabilities of execution of a $DO$ when the order imbalance is of size 2 submitted by either an informed or uninformed trader at $t = 1$ is equal to zero.

[26] Figure IV.4 in the Internet Appendix IV shows a similar figure for a liquid market.

# 4 Market Performance

In this section, we examine how the existence of a $DP$ affects market performance. To do so, in both trading periods we compare the following measures of market quality of the two-venue market in relation to the single-venue market: price informativeness, expected inside spread, expected volume, trade creation, and the expected profits of rational traders.

The signs of the market performance indicators' comparisons may depend on both market and trader characteristics, as well as the trading period, $t$. Recall that these characteristics determine the optimal submission strategy for each type of trader. Hence, in this section, we focus on the transition from the equilibria in the single-venue market to the equilibria in the two-venue market, while in the next section, we focus on the stock market and trader characteristics that lead to these equilibria.

In order to show the signs of the comparisons of the different measures in a compact form, we use the following table format:

| Market quality parameter | $t$ | | | | | |
|---|---|---|---|---|---|---|
| | $E_1^D$ | $E_2^D$ | $E_3^D$ | $E_4^D$ | $E_5^D$ | $E_6^D$ |
| $E_1^{ND}$ | $X$ | | | | $X$ | |
| $E_2^{ND}$ | | $X$ | | | $X$ | $X$ |
| $E_3^{ND}$ | | | $X$ | | $X$ | |
| $E_4^{ND}$ | | | | $X$ | $X$ | $X$ |

The rows of the table show the prevailing equilibria in the single-venue market (Proposition 1), while the columns display the equilibria in the two-venue market (Proposition 2). The cells marked with $X$ show the possible transitions from the prevailing equilibria in the single-venue market to the feasible ones in the two-venue market. In the empty cells, the comparison is not meaningful (as the transition between these equilibria is not possible). The potential symbols in the comparisons are: "=", "<", "  ", ">", and "  ". The sign "=" means that the market quality measure at $t$ is identical; a "<" ( "  " ) shows that the market quality parameter at $t$ corresponding to $E_i^{ND}$ is lower than (lower or equal to) the market quality parameter at $t$ corresponding to $E_j^D$; and the reverse is the case for ">" ( "  " ). The sign " S" means that the result is ambiguous since it depends on the parameters values.

We first study price informativeness, which we can measure in a trading period $t$ by the expected difference between the unconditional variance of the liquidation value of the asset, $\sigma^2$, and the conditional variance of the liquidation value of the asset given the set of prices right after finishing the trading process in which the new trader at $t$ is involved. As Lemma 3 indicates, in the first trading period, only an informed trader might migrate to the $DP$, while in the second trading period, both informed and uninformed traders might trade in this venue. Consequently, the change in the $DP$'s order attractiveness for uninformed traders between the first and the second trading period brings about differences in how the coexistence of the $DP$ with the $LOB$ affects price informativeness in both trading periods, as we show in the next proposition.

**Proposition 3 (Price informativeness)** *The coexistence of a DP with an exchange has the following effects on price informativeness:*

| *Price Informativeness* | $t=1$ | | | | | | $t=2$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $E_1^D$ | $E_2^D$ | $E_3^D$ | $E_4^D$ | $E_5^D$ | $E_6^D$ | $E_1^D$ | $E_2^D$ | $E_3^D$ | $E_4^D$ | $E_5^D$ | $E_6^D$ |
| $E_1^{ND}$ | $=$ | | | | $>$ | | | | | | $>$ | |
| $E_2^{ND}$ | | $=$ | | | $>$ | $>$ | | S | | | $>$ | $>$ |
| $E_3^{ND}$ | | | $=$ | | $>$ | | | | S | | $>$ | |
| $E_4^{ND}$ | | | | $=$ | $>$ | $>$ | | | | S | $>$ | $>$ |

In the first trading period, dark trading harms price informativeness since the $DP$ is only attractive to informed traders. In the second trading period, this result also holds when there is a transition from equilibrium $E_i^{ND}$ to $E_5^D$ or $E_6^D$ or from $E_1^{ND}$ to $E_1^D$. In both these cases, the reduction of the informational content of prices at $t=1$ dominates any potential increase in price informativeness at $t=2$. For example, in $E_1^{ND}$, an uninformed trader at $t=2$ always chooses $NT$, and hence, we never expect an increase in price informativeness when a $DP$ exists.

In all the other instances of the second trading period, we obtain ambiguous results. Let us illustrate this ambiguity with an example. Consider the equilibrium where at $t=1$, an informed trader chooses a $MO$ and an uninformed trader decides $NT$, both in the single-venue and two-venue market; that is, $E_2^{ND}$ and $E_2^D$. When there is no change in traders' behavior at $t=2$, then price informativeness stays the same. However, if at $t=2$ the informed and uninformed traders choose different trading venues, then we have contrasting results regarding price informativeness in the second trading period. If there is segmentation of the order flow such that the informed trader goes to the $DP$, while the uninformed trader remains in the exchange, then we expect a reduction in price informativeness analogous to the first trading period (see Figure 6, Panel a). By contrast, when there is segmentation of the order flow but the informed trader stays in the exchange and the uninformed trader migrates to the $DP$, then we expect an increase in price informativeness in the second trading

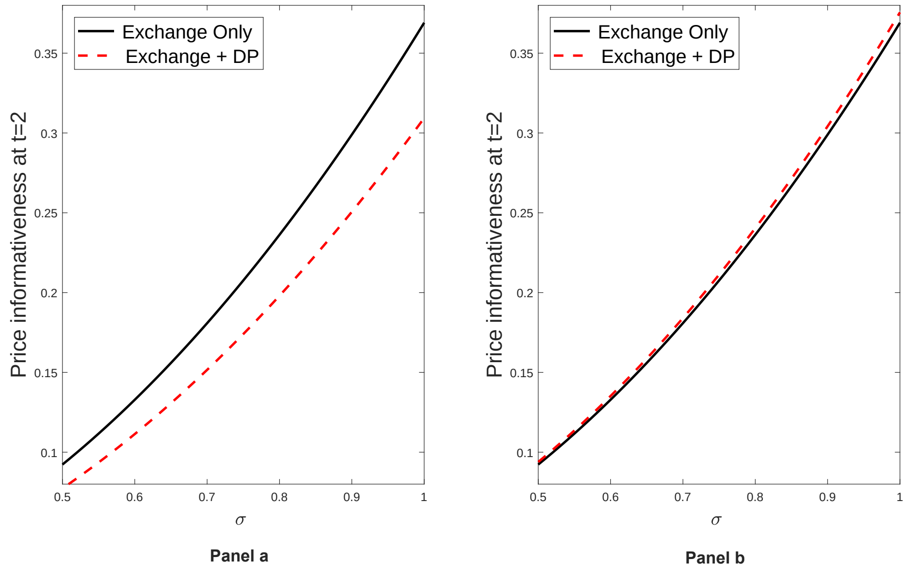


Figure 6: Price informativeness at $t = 2$. Parameters values: $k_1 = 5$, $k_2 = 6$, $\lambda = 0.75$, $\pi = 0.5$, $\tau = 0.05$, and $\delta = 0.9$. In Panel a) the values of $\theta_2^I$ and $\theta_2^U$ are such that only the informed trader goes to the dark pool at $t = 2$. In Panel b) the values of $\theta_2^I$ and $\theta_2^U$ are such that only the uninformed trader goes to the dark pool at $t = 2$.

period (see Figure 6, Panel b). Note that in this example, the stock market and trader characteristics are the same for Panels a) and b), and are such that equilibria $\mathsf{E}_2^{ND}$ and $\mathsf{E}_2^D$ arise. The key variables that determine the traders' behavior and lead to an increase or a decrease in price informativeness in this example are the execution probabilities in the $DP$ at $t = 2$. However, stock market and trader characteristics are also important in the magnitude of this change (for instance, price informativeness increases with fundamental volatility, as does the increase/decrease in price informativeness due to the coexistence of the $DP$ with the exchange).

The next proposition shows how access to a $DP$ affects market liquidity, measured by the ex-ante expected inside spread in the exchange, denoted by $\mathsf{E}_0(S_t)$, in each trading period.

**Proposition 4 (Expected inside spread)** *The existence of a DP alongside the exchange has the following effects on ex-ante expected spreads:*

| $\mathsf{E}_0(S_t)$ | $t=1$ | | | | | | $t=2$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\mathsf{E}_1^D$ | $\mathsf{E}_2^D$ | $\mathsf{E}_3^D$ | $\mathsf{E}_4^D$ | $\mathsf{E}_5^D$ | $\mathsf{E}_6^D$ | $\mathsf{E}_1^D$ | $\mathsf{E}_2^D$ | $\mathsf{E}_3^D$ | $\mathsf{E}_4^D$ | $\mathsf{E}_5^D$ | $\mathsf{E}_6^D$ |
| $\mathsf{E}_1^{ND}$ | $=$ | | | | $>$ | | | | | | $>$ | |
| $\mathsf{E}_2^{ND}$ | | $=$ | | | $>$ | $>$ | | | | | $>$ | $>$ |
| $\mathsf{E}_3^{ND}$ | | | $=$ | | $<$ | | | | | | S | |
| $\mathsf{E}_4^{ND}$ | | | | $=$ | $<$ | $<$ | | | | | $>$ | S |

Proposition 4 indicates that in the first trading period, if the existence of a $DP$ makes the informed trader switch from a $MO$ to a $DO$ (a $LO$ to a $DO$), then the expected inside spread decreases (increases), regardless of the behavior of the uninformed trader. We can explain these results by noting that the switch from a $MO$ to a $DO$ reduces the inside spread, the switch from a $LO$ to a $DO$ increases the inside spread, and the switch from $NT$ to a $LO$ reduces the inside spread. Hence, the change in the informed trader's strategy from a $MO$ to a $DO$ leads to a reduction in the expected inside spread (because the informed trader does not demand anymore liquidity in the exchange), while the uninformed trader either submits the same type of order or switches from $NT$ to a $LO$, which implies that the inside spread remains the same or decreases. Hence, in this case the expected inside spread in the two-venue market is always strictly lower than in the single-venue market. By contrast, the change in the informed trader's strategy from a $LO$ to a $DO$ results in an increase of the expected inside spread, since the informed trader does not supply anymore liquidity in the exchange. In this case, if the uninformed trader does not change his trading strategy, then we unambiguously expect a larger inside spread when investors have access to the $DP$. Otherwise, that is, if the uninformed trader switches from $NT$ to a $LO$, then the change in the expected inside spread is potentially ambiguous. On the one hand, the inside spread increases because the informed trader does not provide any liquidity to the $LOB$; on the other hand, the inside spread might decrease because now the uninformed trader supplies liquidity to the $LOB$. This potential ambiguity arises in the transition from $\mathsf{E}_4^{ND}$ to $\mathsf{E}_5^{D}$. However, note that $\mathsf{E}_4^{ND}$ prevails only if the probability that an informed trader arrives is sufficiently high $\left(\pi > \frac{1}{2}\right)$. Hence, the effect of the informed trader on the inside spread dominates the effect of the uninformed trader, and therefore, the expected inside spread in the two-venue market is unequivocally larger than in the single-venue market. Our results for high fundamental volatility stocks are in line with the conjecture by Buti et al. (2017) that dark trading would not necessarily cause a wider spread even under asymmetric information. However, our results differ for low fundamental volatility stocks.

At the beginning of the second trading period, we could have different spreads depending on whether the $DP$ is available or not. If we have a higher or equal inside spread at the beginning of $t = 2$ in the single-venue market, then having access to the $DP$ unambiguously reduces the ex-ante expected inside spread. This is because at $t = 2$, an informed trader might switch from a $MO$ to a $DO$, and an uninformed trader from a $MO$ or $NT$ to a $DO$, which reduces the expected inside spread. However, if we expect a lower inside spread at the beginning of the second trading period in the single-venue market, then we obtain ambiguous results. The possibility of submitting a $DO$ instead of a $MO$ in the second trading period might reduce the inside spread, which goes in the opposite direction to the one obtained in the first trading period.

We next analyze how the existence of the $DP$ changes the expected trading volume in the exchange, denoted by $\mathsf{E}_0(V_{EX,t})$, and in the $DP$ in both trading periods. From the expected total trading volume in period $t$, denoted by $\mathsf{E}_0(V_{T,t})$, that aggregates the volume in the possible trading venues, we can infer whether trade creation or destruction occurs depending on whether the total expected trading volume increases or decreases, respectively.

**Proposition 5 (Expected volume and trade creation)** *The existence of the DP alongside the exchange has the following effects on:*

*(i) The ex-ante expected volume in the exchange at $t = 1$:*

| $\mathsf{E}_0(V_{EX,1})$ | $\mathsf{E}_1^D$ | $\mathsf{E}_2^D$ | $\mathsf{E}_3^D$ | $\mathsf{E}_4^D$ | $\mathsf{E}_5^D$ | $\mathsf{E}_6^D$ |
|---|---|---|---|---|---|---|
| $\mathsf{E}_1^{ND}$ | $=$ | | | | $>$ | |
| $\mathsf{E}_2^{ND}$ | | $=$ | | | $>$ | $>$ |
| $\mathsf{E}_3^{ND}$ | | | $=$ | | $=$ | |
| $\mathsf{E}_4^{ND}$ | | | | $=$ | $=$ | $=$ |

*At $t = 2$ in the two-venue market, the ex-ante expected volume in the exchange is lower than or equal to the single-venue market if there has not been order migration to the dark at $t = 1$, and it is otherwise ambiguous.*

*(ii) Trade creation or destruction at $t = 1$:*

| $\mathsf{E}_0(V_{T,1})$ | $\mathsf{E}_1^D$ | $\mathsf{E}_2^D$ | $\mathsf{E}_3^D$ | $\mathsf{E}_4^D$ | $\mathsf{E}_5^D$ | $\mathsf{E}_6^D$ |
|---|---|---|---|---|---|---|
| $\mathsf{E}_1^{ND}$ | $=$ | | | | $>$ | |
| $\mathsf{E}_2^{ND}$ | | $=$ | | | $>$ | $>$ |
| $\mathsf{E}_3^{ND}$ | | | $=$ | | $<$ | |
| $\mathsf{E}_4^{ND}$ | | | | $=$ | $<$ | $<$ |

*At $t = 2$, there might be trade creation or destruction in each of the possible equilibria comparisons.*

In the first trading period in the two-venue market, the expected trading volume in the exchange remains the same as in the single-venue market, except if the informed trader switches from a $MO$ to a $DO$. In this last case, the ex-ante expected volume in the exchange decreases. This is because the informed trader's $MO$ migrates to the $DP$, while the uninformed trader either does not change the order type or switches from $NT$ to a $LO$, which does not cause any change in the exchange's expected trading volume at $t = 1$.

In terms of the expected total trading volume, we find that at $t = 1$, the total expected trading volume remains the same if there is no order migration to the $DP$, decreases if the informed trader switches from a $MO$ to a $DO$, and increases if the informed trader switches from a $LO$ to a $DO$.[27] The total expected trading volume decreases when the informed trader switches from a $MO$ to a $DO$ since orders that are submitted to the $DP$ do not execute with certainty. In contrast, when the existence of the $DP$ makes the informed trader switch from a $LO$ to a $DO$, trade creation occurs in this trading period because the expected volume in the $DP$ increases.

In the second trading period in the two-venue market, the expected trading volume in the exchange is less than or equal to the single-venue market case if there has been no order migration to the dark at $t = 1$. This is because at $t = 2$ in the single-venue market, the informed trader chooses a $MO$, while the uninformed trader choose a $MO$ or $NT$. However, in the two-venue market, when market conditions are favorable both traders may also choose to trade in the $DP$. Hence, the expected trading volume in the exchange is either lower or remains unchanged when there is a $DP$. However, when an order has gone to the dark at $t = 1$, then the expected trading volume in the exchange is ambiguous because if the dark order is not executed at $t = 1$ then it returns to the exchange at $t = 2$, thus increasing the expected trading volume in the exchange.

At $t = 2$, we show that it is possible to have trade creation or destruction in each of the possible equilibria comparisons. We discuss the rationale of this ambiguous result. An informed trader in the single-venue market always chooses a $MO$ at $t = 2$, while in the two-venue market, the informed trader could migrate to the $DP$, which destroys trade in the second trading period. An uninformed trader in the single-venue market always chooses a $MO$ or $NT$, while in the two-venue market the uninformed trader might migrate to the $DP$. If this trader changes from a $MO$ to a $DO$, then there is trade destruction, while if there is a switch from $NT$ to a $DO$, there is trade creation at $t = 2$. Hence, if in the two-venue market both informed and uninformed traders choose a $DO$ instead of a $MO$ in some states of the $LOB$, then there is trade destruction. However, we could have trade creation if the uninformed trader switches from $NT$ to a $DO$ and the informed trader's behavior does not vary.

In the next proposition, we compare the unconditional expected profits of rational traders in both trading periods in the two-venue market in relation to the single-venue market. Define $\mathsf{E}_0(\Pi_{t,U})$ as the ex-ante expected profits in trading period $t$ for an uninformed trader.

[27] Note that we consider only trade creation or destruction at $t = 1$. However, if we considered trade creation or destruction across periods, we would find the same amount of trade with or without the $DP$ if the informed trader selected a $MO$. This is because if an informed traders' $MO$ switches to a $DO$ and it is not executed in the $DP$, then it returns as a $MO$ at the end of the second trading period.

Proposition 6 (Expected profits) *(i) In both trading periods, the informed traders' unconditional expected profits are larger or equal in the two-venue market than in the single venue market.*

*(ii) The comparison of uninformed trader's unconditional expected profits is as follows:*

| $E_0(\Pi_{t,U})$ | $t=1$ | | | | | | $t=2$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $E_1^D$ | $E_2^D$ | $E_3^D$ | $E_4^D$ | $E_5^D$ | $E_6^D$ | $E_1^D$ | $E_2^D$ | $E_3^D$ | $E_4^D$ | $E_5^D$ | $E_6^D$ |
| $E_1^{ND}$ | $=$ | | | | | | $<$ | | | | $<$ | |
| $E_2^{ND}$ | | $=$ | | | $<$ | $=$ | | | | | S | S |
| $E_3^{ND}$ | | | $=$ | | | | | | $<$ | | S | |
| $E_4^{ND}$ | | | | $=$ | $<$ | $=$ | | | | $<$ | S | S |

In the first trading period, the expected profits of each type of rational trader are not lower in the two-venue market compared to the single-venue market. First, an informed trader strictly increases his profits since the price improvement obtained by submitting a $DO$ outweighs the execution risk in the $DP$. Second, even if an uninformed trader does not go to the $DP$ at $t=1$, he has larger profits in the two-venue market under certain market conditions. This is because the migration of the informed trader's orders to the $DP$ reduces adverse selection in the $LOB$. Consequently, the probability that an uninformed trader faces an informed trader is smaller in the two-venue market. Therefore, the uninformed trader's expected profits are higher or equal in the two-venue market compared to the single-venue market. Moreover, when the uninformed trader switches from $NT$ to a $LO$, his profits are strictly larger in the two-venue market.

In the second trading period, the informed trader's expected profits are not lower in the two-venue market. However, the changes in the uninformed trader's expected profits are ambiguous, except when market conditions are such that the prevailing equilibrium at $t=1$ is $E_1^{ND}$ in the single-venue market. In this last case, the uninformed trader's expected profits are always larger in the two-venue market case. We next discuss the rationale for the ambiguous cases. By looking at the state of the $LOB$, the uninformed trader can extract information about the value of the asset, which might make the trader choose a $MO$ at $t=2$ in the single-venue market. However, in the two-venue market, the uninformed trader chooses a $DO$ since it offers a better price, but if $\theta_2^U$ is sufficiently low, then these ex-ante expected profits are low, and therefore, the uninformed trader is better off in the single-venue market. Thus, in this situation, it is not beneficial for an uninformed trader to leave the exchange and migrate to the $DP$.

# 5 Discussion

In this section, we highlight some of the implications of our model that may be tested in applied work. These implications are relevant for the current policy and regulatory debate on the effects of dark trading on price informativeness and order flow fragmentation, market liquidity, and trade creation.

The existing empirical research often gives conflicting results on the effects of the presence of the $DP$ alongside an exchange. Studies differ in their research questions, the type of data, and regulatory environments. Thus, most of these empirical studies suggest that the discrepancies are driven by differences in the market structure and financial regulations. Interestingly, our analysis predicts that the coexistence of the two venues may have both negative and positive effects on the market performance of the $LOB$, even if the market structure and regulatory environment are exactly the same. As our previous analysis shows, stock and trader characteristics affect the optimal order submission strategies, and in turn, these have implications for market quality and traders' profits. The rest of the section uses the stock categorization with respect to fundamental asset volatility and information asymmetry defined in Section 2, but we can obtain the same empirical implications with respect to initial stock liquidity, traders' immediacy, or rational traders' participation rate. The predictions in this section are all in relation to the single-venue market in which trading is possible only in the $LOB$.

## Price Informativeness

The first set of implications concerns price informativeness, which is at the heart of the regulatory debate about whether $DP$ increases or decreases price discovery. In the two-venue market, price informativeness is lower in the first trading period if there has been order migration to the $DP$. Its effects at $t = 2$ are ambiguous except for *"High-Low"* stocks (which always lead to a lower or equal price informativeness). For the rest of the cases at $t = 2$ it holds that price informativeness is always lower if an order that was sent to the dark at $t = 1$, and may be higher, lower or equal otherwise.

The implications regarding price informativeness are a consequence of the order flow segmentation results. We find order flow segmentation in the first trading period (only the informed trader sends an order to the $DP$ if conditions are favorable), while in the second trading period, there might or might not be order flow segmentation (since both the informed and uninformed trader might submit orders to the $DP$ if conditions are favorable). The prediction that in the first trading period price informativeness falls due to the presence of the $DP$ is consistent with the existing empirical results of Hendershott and Jones (2005), Comerton-Forde and Putniņš (2015), when the proportion of dark trading is above 10%, Hatheway et al. (2017), and Brogaard and Pan (2019). With regards to the order flow segmentation, Naes and Odegaard (2006) find that there is informational content in crossing network trades, while Nimalendran and Ray (2014) find that informed traders strategically use both crossing networks and exchanges.

Furthermore, our results for the second trading period, when price informativeness may increase due to the existence of a $DP$, are consistent with the empirical evidence reported by Comerton-Forde and Putniņš (2015), who show that for low levels of dark trading, the effects on price discovery are benign or beneficial. For given stock and trader characteristics, the order flow segmentation in the second trading period can be consistent with empirical studies reporting that informed traders concentrate in the exchange while uninformed traders use the $DP$, such as Ready (2014) and Comerton-Forde and Putniņš (2015) (for low levels of dark trading).

### Market liquidity

The second type of implications concerns market liquidity (measured by the expected inside spread in the exchange). An order sent to the $DP$ at $t = 1$ has the following effects: (i) for high fundamental volatility stocks, it leads to a lower expected inside spread in the first and second trading periods; (ii) for low fundamental volatility stocks, it leads to a higher expected inside spread in the first trading period.

Our theoretical results potentially reconcile the mixed empirical results in the existing literature. Several studies show that the existence of a $DP$ decreases market liquidity (Nimalendran and Ray, 2014; Weaver, 2014; Kwan et al., 2015; Degryse et al., 2015; Hatheway et al., 2017). Our analysis predicts that these results are relevant for low fundamental volatility stocks in the first trading period, where an informed trader supplies liquidity in the single-venue model. In this case, the expected spread is higher when this trader migrates to the $DP$. Other studies show that $DP$ trading increases market liquidity (Gresse, 2006; Buti et al., 2011; Ready, 2014). Our paper predicts that this result will occur for high volatility stocks, where an informed trader demands liquidity in the single-venue model, and the expected inside spread is lower when this trader migrates to the $DP$. Finally, Foley and Putniņš (2016) show that midpoint dark trading in the Canadian market does not benefit or harm market liquidity, and Gresse (2017) shows that dark trading is not harmful to any dimension of market liquidity.

Our results are also related to the empirical work that studies how the effects of a $DP$ vary with the tick size. In our model, the classification of high/low fundamental volatility stocks depends on the tick size, among other dimensions. Ceteris paribus, we see that as the tick size increases, the low volatility region expands (where in the first trading period the informed trader submits a limit order when the $DP$ is not available). This result implies that markets with a high tick size are more likely to see an increase in the expected inside spread when there is order migration to the $DP$. This result is similar to that reported by Buti et al. (2015), who show that allowing dark orders to "queue-jump" displayed orders reduces traders' willingness to display $LO$s on competing lit markets. Our results are also consistent with Buti et al. (2011) and Kwan et al. (2015), who show that when spreads on traditional exchanges are constrained by minimum pricing increments, traders have incentives to migrate to a $DP$ since the execution risk in the $DP$ is lower than that for $LO$s in the exchange.[28]

### Trading Volume and Trade Creation

The third set of implications concerns expected volume and trade creation or destruction. In the two-venue market, an order sent to the dark pool in the first trading period leads to the following effects: (i) for high fundamental volatility stocks, the expected trading volume in the exchange and total trading volume is lower in the first trading period; (ii) for low fundamental volatility stocks in the first trading period, the expected trading volume in the exchange does not change and total trading volume is higher.

The previous prediction is related to studies that analyze how stock heterogeneity affects dark trading. Gresse (2006) shows that crossing networks do not attract orders from most illiquid stocks, but do attract orders on stocks that are infrequently traded in the exchange, thus suggesting the possibility that crossing networks might foster trade creation. Furthermore, Menkveld et al. (2017) show that their pecking order theory (the low-cost, low-immediacy venues such as midpoint $DP$s are at the top of the pecking order, whereas the high-cost, high-immediacy venues such as lit exchanges at the bottom) varies with stock characteristics (such as size). We can explain this result in our model by the cut-off execution probability in the $DP$ that generates order migration, which is lower

[28]Note that our results are similar, but the mechanism is different. In our model, the tick size does not affect the execution probability in the $DP$, but it affects the profits obtained in case of execution.

for low volatility, low liquidity stocks and becomes larger for higher volatility, higher liquidity stocks (see Figure 5). Consequently, traders' preference for the lit market is higher for high volatility or high liquidity stocks. Thus, our results are consistent with Menkveld et al. (2017), since we show that the pecking order pattern is weaker if the spread is lower. Similarly, Degryse et al. (2018) find that the higher the stock volatility is, the lower the volume of dark trading is.

## Policy implications

Our work can also inform the regulatory debate on $DP$s. We use our analysis to formulate two additional policy related predictions. First, the European Commission aims to limit dark trading through the Double Volume Cap ($DVC$) mechanism as part of MiFID II/MiFIR, that was implemented in 2018. The $DVC$ introduces a cap on dark trading that limits the trading volume of a financial instrument in any single $DP$ to 4% of its total volume of trading in the previous year. We can think of this cap on dark trading as equivalent to an upper limit on the execution probability in the $DP$. It is worth stressing that if the cap on dark trading is binding such that the informed trader is restricted to trading in the exchange instead of trading in the $DP$ in the first trading period, then the $DVC$ limits both the benefits and the drawbacks of the coexistence of a $DP$ and an exchange. In particular, our analysis implies that imposing a binding cap on dark trading has the following effects on market quality in the first trading period: (i) price informativeness is higher; (ii) the expected inside spread is higher for high fundamental volatility stocks and lower for low fundamental volatility stocks; (iii) trading volume is higher for high fundamental volatility stocks and will remain the same as without the cap for low volatility stocks; (iv) trade creation occurs for high fundamental volatility stocks and trade destruction occurs for low fundamental volatility stocks.

Our results show that while the $DVC$ is expected to have a positive effect on price informativeness in the first trading period, its effects on the other market quality parameters depend on stock and trader characteristics. Specifically, the $DVC$ policy might have unintended negative consequences, such as decreasing liquidity for high volatility stocks, trade destruction for low volatility stocks, or decreasing the profits for rational traders. To the best of our knowledge, the only study that examines the consequences of $DVC$ is Johann et al. (2019). They support our view that the consequences of the ban may not be the ones that regulators expected. However, they note that with the implementation of the ban, trading volume migrated from $DP$s to "quasi-dark" trading mechanisms rather than back to exchange markets (three times more volume went to these "quasi dark venues", which are alternative venues that do not exist in our model). They also find a negligible impact of $DP$ caps on market liquidity and short-term price efficiency.

Second, if we interpret that informed traders in our model are high frequency traders ($HFT$), as in Biais et al. (2015) and Foucault et al. (2016), we may address a very controversial restriction related to the presence of $HFT$s in the $DP$s, and therefore, study the impact of limiting $HFT$s' access to the $DP$ on investors' profits. $HFT$s have a speed and technology advantage in relation to other traders, and as such, they can trade ahead of other traders. Many $DP$s advertise that they provide investors with protection against predatory $HFT$ trading.[29] Restricting informed traders' access to the $DP$ leads to exactly the same equilibria as in the case when the probability of execution for the informed trader in the $DP$ in the first trading period is very small, so the informed trader remains in the exchange. Again, both the benefits and drawbacks of introducing a $DP$ would disappear after imposing this type of restriction. We show that under some market conditions, limiting $HFT$ trading in the $DP$ negatively affects the profits of all types of traders. Thus, it does not increase the uninformed trader's profits in the first trading period, and decreases this trader's profits for *"High-Low"* stocks in the second trading period.

To sum up, the empirical and policy implications we derive from our paper show a clear need to study the interaction between a $DP$ and a $LOB$ in more depth since stock and trader characteristics have a fundamental effect. Moreover, restricting informed trading in the $DP$ may have unintended consequences.

## 6 Concluding Remarks

This paper examines the impact of an opaque dark pool that competes with a transparent exchange organized as a limit order book in a two-period model with asymmetric information about the liquidation value of the asset. In the first trading period, we find that only the informed trader diverts his order from the exchange to the dark pool, provided that the execution risk in the dark pool is

[29]See the New York Attorney General versus Barclays case for misconduct of a Barclays' $DP$. The US SEC Commissioner Luis Aguilar (2015) remarked that "dark pools initially portrayed themselves as havens from predatory traders. They achieved this, in part, by excluding high frequency traders, who supposedly use brute speed to front-run institutional investors' large orders. Lured by this promise of safety, institutional traders embraced ATSs as a solution to their trading needs. Unfortunately, all too often the safety these investors sought proved illusory." Interestingly, in the case of the misconduct of Barclays' $DP$, "Liquidity Cross", Barclays allegedly encouraged HFTs to use its pool to increase their activity in the $DP$ and thereby increase liquidity.

sufficiently low. However, in the next trading period, both informed and uninformed traders may trade in the dark pool if conditions are favorable.

Our results show how the competition of a dark pool with an exchange affects price informativeness and market performance. We find that the results depend critically on the stock categorization in terms of high/low volatility and high/low information asymmetry as these factors are determinant when selecting the venue and the type of order. Furthermore, we have discussed how these results are related to previous empirical work. Moreover, our policy analysis concludes that regulators should consider that establishing measures to reduce informed traders' participation in dark pools could have unintended negative consequences on other traders and on some market performance indicators.

Future work could extend our theoretical model in different ways, such as by considering the case in which the initial prices of the limit order book are asymmetrically located in the grid. Allowing for this asymmetry might induce an uninformed trade to migrate to the dark pool initially. Another interesting theoretical generalization of our model may include modelling the execution probability in the dark as an increasing function of fundamental volatility or fully endogenizing the behavior of the dark pool liquidity provider. Modelling the execution probability in the dark pool as an increasing function of fundamental volatility (as the empirical literature suggests) seems not to affect the existence of the equilibria. However, it varies the thresholds at which all traders execute their orders in the exchange and those equilibria where there is migration to the dark pool. When the execution probability does not depend on volatility, a market order is more desirable for informed traders as a high volatility increases their informational advantage. However, if the execution probability in the dark pool increases also with fundamental volatility, the dark order becomes more profitable for high volatility stocks (relative to the constant case). Additionally, our results call for more empirical and experimental work to test the predictions of our model, and more generally, for the development of applied work studying the effects of asymmetric information in the competition between trading venues with different degrees of transparency on market quality and traders' profits.

# Appendices

## A Notation summary

This appendix summarizes the key notations used in the paper.

**Types of Traders**

| Type | Definition |
|---|---|
| $R$ | Rational trader, $R \in \{I, U\}$ |
| $I$ $(IH/IL)$ | Informed trader (who observes a high/low liquidation value) |
| $U$ $(UB/US)$ | Uninformed trader (who buys/sells) |
| $LT$ | Liquidity trader |

**Types of Orders**

| Type | Definition |
|---|---|
| $MO$ $(BMO/SMO)$ | Market order (Buy/Sell market order) |
| $LO$ $(BLO/SLO)$ | Limit order (Buy/Sell limit order) |
| $DO$ $(BDO/SDO)$ | Dark order (Buy/Sell dark order) |
| $NT$ | No trade |

**Exogenous Parameters**

| Parameters | Definition |
|---|---|
| $\widetilde{V}$ | Liquidation value of the asset, may take two values $V \in \{V^H, V^L\}$ |
| $\mu$ and $\sigma$ | The unconditional mean and volatility of the liquidation value $\widetilde{V}$ |
| $A_1^p$, $B_1^p$ | Ask and bid prices at time $t = 1$ and position $p$ |
| $\tau$ | Tick size |
| $k_p$ | A natural number such that $A_1^p = \mu + k_p\tau$ ($B_1^p = \mu - k_p\tau$) |
| $\kappa$ | A real number such that $\sigma = \kappa\tau$ |
| $\lambda$ | Probability that a rational trader arrives to the market |
| $\pi$ | Probability that a rational trader is informed |
| $PIN$ | Probability that an informed trader arrives to the market which is equal to $\pi\lambda$ |
| $\delta$ | Discount factor (immediacy) of rational traders |
| $\theta_1^R$ | Probability of execution of a $DO$ at $t = 1$ for a rational trader |

**Endogenous Variables**

| Variable | Definition |
|---|---|
| $A_2^p$, $B_2^p$ | Ask and bid prices at time $t = 2$ and position $p$ |
| $\theta_2^R$ | Probability of execution of a $DO$ at $t = 2$ for a rational trader |
| $\Pi_{O,t}^R$ | Profit for a trader of type $R$ using order $O$ at date $t$ |
| $\kappa_{MO-LO}^I$ | Volatility threshold for informed trader's decision between $MO$ and $LO$ |
| $\psi_{LO-NT}^U$ | $PIN$ threshold for uninformed trader's decision between $LO$ and $NT$ |

In addition, $LOB$ denotes the limit order book, $DP$ denotes the dark pool, $ND$ denotes the single-venue market, and $D$ denotes the two-venue market.

## B Single-venue market model

**Definition B.1** *Let us define $\Omega_o$ and $\Gamma_o$ as the probability that an informed trader and uninformed trader at $t = 1$ choose an order $O\ 2\ O_{ND}$, where $o = 0$ corresponds to a NT order; $o = 1$ to a MO; $o = 2$ to a LO; and such that $\sum_{o=0}^{2} \Omega_o = 1$ and $\sum_{o=0}^{2} \Gamma_o = 1$.*

**Proof of Lemma 1.** We solve the game backwards. At $t = 2$, the expected profits for an informed and uninformed buyer and seller are:

| State of the book | IH | | | IL | | |
|---|---|---|---|---|---|---|
| | $BMO$ | $BLO$ | $NT$ | $SMO$ | $SLO$ | $NT$ |
| $(A_1^1, B_1^1)$ | $(\kappa \quad k_1)\,\tau$ | 0 | 0 | $(\kappa \quad k_1)\,\tau$ | 0 | 0 |
| $(A_1^2, B_1^1)$ | $(\kappa \quad k_2)\,\tau$ | 0 | 0 | $(\kappa \quad k_1)\,\tau$ | 0 | 0 |
| $(A_1^1, B_1^1 + \tau)$ | $(\kappa \quad k_1)\,\tau$ | 0 | 0 | $(\kappa \quad k_1 + 1)\,\tau$ | 0 | 0 |
| $(A_1^1, B_1^2)$ | $(\kappa \quad k_1)\,\tau$ | 0 | 0 | $(\kappa \quad k_2)\,\tau$ | 0 | 0 |
| $(A_1^1 \quad \tau, B_1^1)$ | $(\kappa \quad k_1 + 1)\,\tau$ | 0 | 0 | $(\kappa \quad k_1)\,\tau$ | 0 | 0 |

Table B.1: Expected profits of an informed buyer ($IH$) and an informed seller ($IL$) at $t = 2$ when traders do not have access to the dark pool.

| State of the book | UB | | | UB | | |
|---|---|---|---|---|---|---|
| | $BMO$ | $BLO$ | $NT$ | $SMO$ | $SLO$ | $NT$ |
| $(A_1^1, B_1^1)$ | $k_1\tau$ | 0 | 0 | $k_1\tau$ | 0 | 0 |
| $(A_1^2, B_1^1)$ | $(X\kappa \quad k_2)\,\tau$ | 0 | 0 | $(k_1 + X\kappa)\,\tau$ | 0 | 0 |
| $(A_1^1, B_1^1 + \tau)$ | $(Y\kappa \quad k_1)\,\tau$ | 0 | 0 | $(k_1 \quad 1 + Y\kappa)\,\tau$ | 0 | 0 |
| $(A_1^1, B_1^2)$ | $(X\kappa + k_1)\,\tau$ | 0 | 0 | $(X\kappa \quad k_2)\,\tau$ | 0 | 0 |
| $(A_1^1 \quad \tau, B_1^1)$ | $(Y\kappa + k_1 \quad 1)\,\tau$ | 0 | 0 | $(Y\kappa \quad k_1)\,\tau$ | 0 | 0 |

Table B.2: Expected profits of an uninformed buyer ($UB$) and an uninformed seller ($US$) at $t = 2$ when traders do not have access to the dark pool.

Note that at $t = 2$ the expected profits of each strategy depend on the state of the $LOB$ (which depends on the chosen strategy at $t = 1$). Uniformed traders at $t = 2$ form beliefs about the strategies and type of player at $t = 1$. Thus, we define the uninformed traders' belief at $t = 2$ about the probability that the $MO$ (observed in the $LOB$) was submitted by an informed trader as

$$X = \frac{\lambda\pi\Omega_1}{1 \quad \lambda + \lambda\pi\Omega_1 + \lambda\,(1 \quad \pi)\,\Gamma_1}. \tag{B.1}$$

Similarly, we define the uninformed traders' belief at $t = 2$ about the probability that the $LO$ (observed in the $LOB$) was submitted by an informed trader as

$$Y = \frac{\pi\Omega_2}{\pi\Omega_2 + (1 \quad \pi)\,\Gamma_2}. \tag{B.2}$$

By comparing the expected profits of an informed trader at $t = 2$ we obtain that the informed trader always submits a $MO$. Similarly, we compare the profits of an uniformed trader and see that he never chooses to submit a $LO$. His choice between a $MO$ or $NT$ depends on the uninformed trader beliefs that the order placed at $t = 1$ that he observes in the book comes from an informed trader, as it can be seen in Table B.3.

| State of the book | UB | US |
|---|---|---|
| $(A_1^1, B_1^1)$ | $NT$ | $NT$ |
| $(A_1^2, B_1^1)$ | $\begin{cases} MO & \text{if } X\kappa > k_2 \\ NT & \text{if } X\kappa \quad k_2 \end{cases}$ | $NT$ |
| $(A_1^1, B_1^1 + \tau)$ | $\begin{cases} MO & \text{if } Y\kappa > k_1 \\ NT & \text{if } Y\kappa \quad k_1 \end{cases}$ | $NT$ |
| $(A_1^1, B_1^2)$ | $NT$ | $\begin{cases} MO & \text{if } X\kappa > k_2 \\ NT & \text{if } X\kappa \quad k_2 \end{cases}$ |
| $(A_1^1 \quad \tau, B_1^1)$ | $NT$ | $\begin{cases} MO & \text{if } Y\kappa > k_1 \\ NT & \text{if } Y\kappa \quad k_1 \end{cases}$ |

Table B.3: Optimal trading strategies of an uniformed buyer ($UB$) and seller ($US$) at $t = 2$ when traders do not have access to the dark pool.

At $t = 1$, the expected profits of an informed and uniformed trader are presented in Table B.4 and Table B.5, respectively.

| IH | IL | Expected Profits |
|---|---|---|
| $BMO$ | $SMO$ | $(\kappa \quad k_1)\,\tau$ |
| $BLO$ | $SLO$ | $\delta\frac{1 \quad \lambda}{2}(\kappa + k_1 \quad 1)\,\tau$ |
| $NT$ | $NT$ | $0$ |

Table B.4: Expected profits of an informed buyer ($IH$) and seller ($IL$) at $t = 1$ when traders do not have access to the dark pool.

| UB | US | Expected Profits |
|---|---|---|
| $BMO$ | $SMO$ | $k_1\tau$ |
| $BLO$ | $SLO$ | $\frac{\delta}{2}((1 \quad \lambda + \lambda\pi)(k_1 \quad 1) \quad \lambda\pi\kappa)\,\tau$ |
| $NT$ | $NT$ | $0$ |

Table B.5: Expected profits of an uninformed buyer ($UB$) and seller ($US$) at $t = 1$ when traders do not have access to the dark pool.

It can be easily seen that at $t = 1$ the informed trader never chooses $NT$, while the uninformed never chooses a $MO$. ■

**Proof of Proposition 1.** We follow the steps outlined in Internet Appendix II to check if a particular strategy profile constitutes a $PBE$.

Because of the symmetry of the model, without any loss of generality, at $t = 1$ we focus on buyers. We distinguish two cases: Case A ($k_1 > 1$) and Case B ($k_1 = 1$).

Case A. We present the full proof for one of the possible strategy profile at $t = 1$ that yields an equilibrium. The proofs of all the other 3 equilibria are sketched here and can be obtained on request from the authors.

$E_1^{ND}$: $(BMO, SMO, BLO, SLO)$

First step. In this case $\Omega_0 = 0$, $\Omega_1 = 1$, $\Omega_2 = 0$, $\Gamma_0 = 0$, $\Gamma_1 = 0$, and $\Gamma_2 = 1$.

Second step. Using Bayes' rule we obtain that $X^{1,ND} = \dfrac{\lambda\pi}{1 \quad \lambda + \lambda\pi}$ and $Y^{1,ND} = 0$.

Third step. Applying Lemma 1, we know that at $t = 2$ the optimal strategy of informed traders is to choose a $MO$, while the optimal strategy of the uninformed trader is as follows:

| State of the book | UB | US |
|---|---|---|
| $(A_1^1, B_1^1)$ | $NT$ | $NT$ |
| $(A_1^2, B_1^1)$ | $\begin{cases} MO & \text{if } \frac{\lambda\pi}{1 \quad \lambda + \lambda\pi}\kappa > k_2 \\ NT & \text{if } \frac{\lambda\pi}{1 \quad \lambda + \lambda\pi}\kappa \quad k_2 \end{cases}$ | $NT$ |
| $(A_1^1, B_1^1 + \tau)$ | $NT$ | $NT$ |
| $(A_1^1, B_1^2)$ | $NT$ | $\begin{cases} MO & \text{if } \frac{\lambda\pi}{1 \quad \lambda + \lambda\pi}\kappa > k_2 \\ NT & \text{if } \frac{\lambda\pi}{1 \quad \lambda + \lambda\pi}\kappa \quad k_2 \end{cases}$ |
| $(A_1^1 \quad \tau, B_1^1)$ | $NT$ | $NT$ |

Table B.6: Optimal responses of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BMO, SMO, BLO, SLO)$.

**Fourth step.** Given the optimal response of traders at $t = 2$, we find the optimal action for all rational traders at $t = 1$.

Informed traders at $t = 1$ have no incentives to deviate from the prescribed strategy profile whenever

$$\kappa \quad k_1 \quad \delta\frac{1 \quad \lambda}{2}(\kappa + k_1 \quad 1). \tag{B.3}$$

Uninformed traders at $t = 1$ have no incentives to deviate from the prescribed strategy if and only if

$$(1 \quad \lambda)(k_1 \quad 1) \quad \lambda\pi(\kappa \quad (k_1 \quad 1)) > 0. \tag{B.4}$$

**Fifth step.** Nobody at $t = 1$ has unilateral incentives to deviate from $(BMO, SMO, BLO, SLO)$ when both conditions $(B.3)$ and $(B.4)$ are satisfied, and these conditions can be rewritten as

$$\kappa^I_{MO \quad LO}\tau \quad \sigma \text{ and } PIN < \psi^U_{LO \quad NT}, \tag{B.5}$$

where the expression of $\psi^U_{LO \quad NT}$ and $\kappa^I_{MO \quad LO}$ are given in the statement of this proposition.

Finally, combining Table B.6 and Expression $(B.4)$, it follows that an uninformed trader always selects $NT$ at $t = 2$.

$E_2^{ND}$: $(BMO, SMO, NT, NT)$

Following the same procedure as above and noting that $X^{2,ND} = \frac{\lambda\pi}{1 \quad \lambda + \lambda\pi}$ and $Y^{2,ND}$ is undetermined $Y^{2,ND}\ 2\ [0, 1]$, we obtain that no trader at $t = 1$ has unilateral incentives to deviate in $E_2^{ND}$ whenever:

$$\kappa \quad k_1 \qquad \delta\frac{1 \quad \lambda}{2}(\kappa + k_1 \quad 1) \text{ and} \tag{B.6}$$

$$0 \qquad (1 \quad \lambda)(k_1 \quad 1) \quad \lambda\pi(\kappa \quad (k_1 \quad 1)), \tag{B.7}$$

which can be rewritten as $\kappa^I_{MO \quad LO}\tau \quad \sigma$ and $PIN \quad \psi^U_{LO \quad NT}$.

Finally, in the following table we include the moves that are in the equilibrium path at $t = 2$ for an uninformed trader, taking into account that $(BMO, SMO, NT, NT)$ is the strategy profile chosen at $t = 1$.

| State of the book | UB | US |
|---|---|---|
| $(A_1^1, B_1^1)$ | $NT$ | $NT$ |
| $(A_1^2, B_1^1)$ | $\begin{cases} MO & \text{if } \frac{\lambda\pi}{1-\lambda+\lambda\pi}\kappa > k_2 \\ NT & \text{if } \frac{\lambda\pi}{1-\lambda+\lambda\pi}\kappa \leq k_2 \end{cases}$ | $NT$ |
| $(A_1^1, B_1^2)$ | $NT$ | $\begin{cases} MO & \text{if } \frac{\lambda\pi}{1-\lambda+\lambda\pi}\kappa > k_2 \\ NT & \text{if } \frac{\lambda\pi}{1-\lambda+\lambda\pi}\kappa \leq k_2 \end{cases}$ |

Table B.7: Optimal choice of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BMO, SMO, NT, NT)$

$\mathsf{E}_3^{ND}$: $(BLO, SLO, BLO, SLO)$

Following the same procedure as above and noting that $X^{3,ND} = 0$ and $Y^{3,ND} = \pi$, we obtain that no trader at $t = 1$ has unilateral incentives to deviate in $\mathsf{E}_3^{ND}$ whenever:

$$\delta\frac{1-\lambda}{2}(\kappa + k_1 - 1) > \kappa - k_1 \text{ and} \tag{B.8}$$

$$(1-\lambda)(k_1 - 1) - \lambda\pi(\kappa - (k_1 - 1)) > 0, \tag{B.9}$$

which can be rewritten as $\sigma < \kappa^I_{MO-LO}\tau$ and $PIN < \psi^U_{LO-NT}$.

Finally, in the following table we include the moves that are in the equilibrium path at $t = 2$ for an uninformed trader, taking into account the conditions that must be satisfied if $(BLO, SLO, BLO, SLO)$ is the strategy profile chosen at $t = 1$.

| State of the book | UB | US |
|---|---|---|
| $(A_1^2, B_1^1)$ | $NT$ | $NT$ |
| $(A_1^1, B_1^1 + \tau)$ | $\begin{cases} MO & \text{if } \pi\kappa > k_1 \\ NT & \text{if } \pi\kappa \leq k_1 \end{cases}$ | $NT$ |
| $(A_1^1, B_1^2)$ | $NT$ | $NT$ |
| $(A_1^1 - \tau, B_1^1)$ | $NT$ | $\begin{cases} MO & \text{if } \pi\kappa > k_1 \\ NT & \text{if } \pi\kappa \leq k_1 \end{cases}$ |

Table B.8: Optimal choice of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BLO, SLO, BLO, SLO)$

$\mathsf{E}_4^{ND}$: $(BLO, SLO, NT, NT)$

Following the same procedure as above and noting that $X^{4,ND} = 0$ and $Y^{4,ND} = 1$, we obtain that no trader at $t = 1$ has unilateral incentives to deviate in $\mathsf{E}_4^{ND}$ whenever:

$$\delta\frac{1-\lambda}{2}(\kappa + k_1 - 1) > \kappa - k_1 \text{ and} \tag{B.10}$$

$$0 \geq (1-\lambda)(k_1 - 1) - \lambda\pi(\kappa - (k_1 - 1)), \tag{B.11}$$

which can be rewritten as $\sigma < \kappa^I_{MO-LO}\tau$ and $PIN \geq \psi^U_{LO-NT}$.

Finally, in the following table we include the moves that are in the equilibrium path at $t = 2$ for an uninformed trader, taking into account the conditions that must be satisfied if $(BLO, SLO, NT, NT)$ is the strategy profile chosen at $t = 1$.

| State of the book | UB | US |
|---|---|---|
| $(A_1^1, B_1^1)$ | $NT$ | $NT$ |
| $(A_1^2, B_1^1)$ | $NT$ | $NT$ |
| $(A_1^1, B_1^1+\tau)$ | $MO$ | $NT$ |
| $(A_1^1, B_1^2)$ | $NT$ | $NT$ |
| $(A_1^1-\tau, B_1^1)$ | $NT$ | $MO$ |

Table B.9: Optimal choice of uninformed traders at $t=2$ when the strategy profile at $t=1$ is $(BLO, SLO, NT, NT)$.

**Case B.** We have to replace $k_1 = 1$ in the proof of Case A. It should only be noted that when $k_1 = 1$ the conditions $(B.4)$ and $(B.9)$ are never satisfied and, therefore, the strategy profiles at $t = 1$ $(BMO, SMO, BLO, SLO)$ and $(BLO, SLO, BLO, SLO)$ cannot be part of an equilibrium of the game. By contrast, when $k_1 = 1$, the conditions $(B.7)$ and $(B.11)$ are always satisfied. However, the condition $(B.10)$ is never satisfied when $k_1 = 1$, and therefore, the strategy profile $(BLO, SLO, NT, NT)$ cannot be either part of an equilibrium of the game.

## C Two-venue market model

**Definition C.1** *Let us define $\Omega_o$ and $\Gamma_o$ as the probability that an informed trader and uninformed trader at $t=1$ choose an order $O \in \mathcal{O}_D$, where $o=0$ corresponds to a $NT$ order; $o=1$ to a $MO$; $o=2$ to a $LO$; $o=3$ to a $DO$; and such that $\sum_{o=0}^{3}\Omega_o = 1$ and $\sum_{o=0}^{3}\Gamma_o = 1$.*

**Definition C.2** *We define as $\mathcal{B}_1$ the set of all possible states of the LOB at the end of the first trading period and by $B_1 \in \mathcal{B}_1$ a possible state of the book (see Internet Appendix I for a full definition). The state of the book $B_1 = \emptyset$ is the state when the best prices in the book are $(A_1^1, B_1^1)$.*

**Proof of Lemma 2.** Let $\widetilde{z}$ be a random variable representing the order imbalance of the $DP$ at the beginning of $t=1$. Suppose that $\widetilde{z}$ is the same for all rational traders (informed and uninformed traders). At $t=1$, for a rational trader $R$, $R = I, U$, let $\theta_1^{R,1} = pr_R(\widetilde{z} \geq 1) = pr_R(\widetilde{z} \leq -1)$ and $\theta_1^{R,2} = pr_R(\widetilde{z} \geq 2) = pr_R(\widetilde{z} \leq -2)$. In particular, $\theta_1^{R} \leq \theta_1^{R,1}$, for $R = I, U$. Applying Bayes' rule, it follows that

$$\begin{aligned}
\theta_2^I &= \lambda\left(\pi\Omega_3 + \frac{(1-\pi)\Gamma_3}{2}\right)\theta_1^{I,2} + \left(1-\lambda\left(\pi\Omega_3 + \frac{(1-\pi)\Gamma_3}{2}\right)\right)\theta_1^{I,1} \text{ and} \\
\theta_2^U &= \lambda\left(\frac{\pi\Omega_3}{2} + \frac{(1-\pi)\Gamma_3}{2}\right)\theta_1^{U,2} + \left(1-\lambda\left(\frac{\pi\Omega_3}{2} + \frac{(1-\pi)\Gamma_3}{2}\right)\right)\theta_1^{U,1}.
\end{aligned}$$

Hence, we have that if the probabilities of execution of $DO$ at $t=1$ coincide for all rational traders ($\theta_1^{I,2} = \theta_1^{U,2}$ and $\theta_1^{I,1} = \theta_1^{U,1}$), then $\theta_2^U - \theta_2^I = \Omega_3\frac{\lambda\pi}{2}(\theta_1^{I,1} - \theta_1^{I,2}) \geq 0$ (with strict inequality when $\Omega_3 \neq 0$). ∎

**Proof of Lemma 3.** Note that the set of the possible states of the $LOB$ is the same as in the case there is no $DP$. However, the state of the book $(A_1^1, B_1^1)$ can be obtained either because a trader arrived and decided not to trade, or because a trader arrived and he submitted a $DO$.

We solve the model backwards. At $t=2$ the expected profits of each strategy depend on the state of the $LOB$. Additionally, uninformed traders form beliefs about the strategies that have been chosen at $t=1$. Let $X$ and $Y$ be defined as in (B.1) and (B.2), respectively, and $Z$ denote the uninformed trader's belief at $t=2$ about the probability that a $DO$ that returns to the exchange as a $MO$ at the end of the second trading period was submitted by an informed, which is equal to

$$Z = \frac{(1-\theta_1^I)\pi\Omega_3}{(1-\theta_1^I)\pi\Omega_3 + (1-\theta_1^U)(1-\pi)\Gamma_3}. \tag{C.1}$$

As in the case when the $DP$ was not available, and without loss of generality, we will focus on the expected profits for an informed and an uniformed buyer at $t = 2$, as summarized in Table C.1 and Table C.2, respectively.

Define $P_I$ as the probability of execution of a limit order placed by an informed trader at $t = 2$ conditional on the fact that there is no change in the $LOB$ during the first trading period, and equals

$$P_I = p_{BLO,2}^{IH}(B_1 = ?) = \frac{(1-\theta_1^U)\frac{1-\pi}{2}\Gamma_3}{\pi\Omega_3 + (1-\pi)(\Gamma_0 + \Gamma_3)}.$$

| **IH** | $BMO$ | $BDO$ | $BLO$ | $NT$ |
|---|---|---|---|---|
| $(A_1^1, B_1^1)$ | $(\kappa - k_1)\tau$ | $\theta_2^I\kappa\tau$ | $P_I\delta(k_1+\kappa-1)\tau$ | 0 |
| $(A_1^2, B_1^1)$ | $(\kappa - k_2)\tau$ | $\theta_2^I\left(\kappa - \frac{k_2-k_1}{2}\right)\tau$ | 0 | 0 |
| $(A_1^1, B_1^1+\tau)$ | $(\kappa - k_1)\tau$ | $\theta_2^I\left(\kappa - \frac{1}{2}\right)\tau$ | 0 | 0 |
| $(A_1^1, B_1^2)$ | $(\kappa - k_1)\tau$ | $\theta_2^I\left(\kappa + \frac{k_2-k_1}{2}\right)\tau$ | 0 | 0 |
| $(A_1^1-\tau, B_1^1)$ | $(\kappa - k_1 + 1)\tau$ | $\theta_2^I\left(\kappa + \frac{1}{2}\right)\tau$ | 0 | 0 |

Table C.1: Expected profits of an informed buyer $(IH)$ at $t = 2$

Define $P_U$ as the probability of execution of a limit order placed by an uninformed trader at $t = 2$ given that there are no changes in prices in the $LOB$ during the first trading period, and equals

$$P_U = p_{BLO,2}^{UB}(B_1 = ?) = \frac{1}{2}\frac{(1-\theta_1^I)\pi\Omega_3 + \left(1-\theta_1^U\right)(1-\pi)\Gamma_3}{\pi\Omega_3 + (1-\pi)(\Gamma_0+\Gamma_3)}.$$

| **UB** | $BMO$ | $BDO$ | $BLO$ | $NT$ |
|---|---|---|---|---|
| $(A_1^1, B_1^1)$ | $k_1\tau$ | 0 | $P_U\delta(k_1 - Z\kappa - 1)\tau$ | 0 |
| $(A_1^2, B_1^1)$ | $(X\kappa - k_2)\tau$ | $\theta_2^U\left(X\kappa - \frac{k_2-k_1}{2}\right)\tau$ | 0 | 0 |
| $(A_1^1, B_1^1+\tau)$ | $(Y\kappa - k_1)\tau$ | $\theta_2^U\left(Y\kappa - \frac{1}{2}\right)\tau$ | 0 | 0 |
| $(A_1^1, B_1^2)$ | $(X\kappa + k_1)\tau$ | $\theta_2^U\left(X\kappa - \frac{k_2-k_1}{2}\right)\tau$ | 0 | 0 |
| $(A_1^1-\tau, B_1^1)$ | $(Y\kappa + k_1 - 1)\tau$ | $\theta_2^U\left(Y\kappa - \frac{1}{2}\right)\tau$ | 0 | 0 |

Table C.2: Expected profits of an uninformed buyer $(UB)$ at $t = 2$

At $t = 1$ the expected profits of an informed $IH$ and an uniformed buyer $UB$ are summarized in Table C.3 and Table C.4, respectively.[30]

| **IH** | **Expected Profits** |
|---|---|
| $BMO$ | $(\kappa - k_1)\tau$ |
| $BLO$ | $\frac{\delta(1-\lambda)}{2}(\kappa + k_1 - 1)\tau$ |
| $BDO$ | $\theta_1^I\kappa\tau + (1-\theta_1^I)\delta\left(\lambda\frac{(1-\pi)}{2}I_{SLO,2}^{US,B_1=?} + (\kappa - k_1) - (k_2 - k_1)\left(\lambda\pi I_{BMO,2}^{IH,B_1=?} + \frac{1-\lambda}{2}\right)\right)\tau$ |
| $NT$ | 0 |

Table C.3: Expected profits of an informed buyer $(IH)$ at $t = 1$

where $I_{SLO,2}^{US,B_1=?}$ and $I_{BMO,2}^{IH,B_1=?}$ are indicator functions such that $I_{SLO,2}^{US,B_1=?} = 1$ if at $t = 2$, an $US$ selects a $SLO$ when the $LOB$ has not changed at $t = 1$, and $I_{SLO,2}^{US,B_1=?} = 0$, otherwise. Similarly, the

[30] Notice that due to the symmetry of the game, the expected profits of the informed $IL$ trader and uninformed seller $US$ are the same as the ones displayed in Tables C.3 and C.4, respectively.

| UB | Expected Profits |
|---|---|
| $BMO$ | $k_1\tau$ |
| $BLO$ | $\frac{\delta}{2}\Big((1 \quad \lambda)(k_1 \quad 1) \quad \lambda\pi I^{IL,B_1=BLO}_{SMO,2}(\kappa \quad k_1+1)\Big)\tau$ |
| $BDO$ | 0 |
| $NT$ | 0 |

Table C.4: Expected profits of an uninformed buyer ($UB$) at $t = 1$

remaining indicator functions can be defined. By simple inspection of the payoffs in Table C.3, it can be seen that an informed buyer at $t = 1$ never chooses $NT$ because it is dominated by a $MO$.

Notice also that the expected profits of a $BDO$ submitted by an informed buyer at $t = 1$ may be positive, and as a result the informed may choose to place a $BDO$ at $t = 1$ depending on how high the execution probability $\theta^I_1$ is null. However, the payoff at $t = 1$ of the $BDO$ for the uninformed trader is always null (see Internet Appendix I). Therefore, we have that $\Gamma_3 = 0,$ and hence $B_1 = ?$ implies either $\Omega_3$ or $\Gamma_0$ is not null. Thus,

$$P_I = p^{IH}_{BLO,2}\,(B_1 = \,?\,) = p^{IL}_{SLO,2}\,(B_1 = \,?\,) = 0.$$

Consequently, informed traders never choose a $LO$ at $t = 2$, since this order is dominated by a $MO$. Uninformed traders also do not select a $LO$ at $t = 2$. To see this, note that Table C.2 shows that we have to prove the result when prices do not change. In such a case we distinguish two cases: $\Omega_3 = 1$ and $\Omega_3 = 0$. In the first case, $Z = 1$ and, therefore, the expected profits of a $LO$ are negative, as shown in Table C.2. In the second case, $B_1 = ?$ due to $\Gamma_0 = 1$. Hence,

$$P_U = p^{UB}_{BLO,2}\,(B_1 = \,?\,) = p^{US}_{SLO,2}\,(B_1 = \,?\,) = 0,$$

and therefore, at $t = 2$ the expected profits of a $LO$ for an uninformed trader are always null.

Let us determine next the optimal strategy for each rational trader at $t = 2$. Depending on the values of the parameters, we have 6 possible cases for the informed trader and 16 cases for the uninformed trader. Due to limits of the length of the paper, the optimal responses of uninformed traders at $t = 2$ will be specified in each equilibria (see proof of Lemma C.1), with

$$\theta_X \qquad \frac{X\kappa \quad k_2}{X\kappa \quad \frac{k_2 \quad k_1}{2}}, \text{ and}$$

$$\theta_Y \qquad \frac{Y\kappa \quad k_1}{Y\kappa \quad \frac{1}{2}}.$$

Next, we focus on informed traders. Given that $\kappa > k_2 > k_1 \quad 1$, the following inequalities hold:

$$\frac{\kappa \quad k_2}{\kappa \quad \frac{k_2 \quad k_1}{2}} < \frac{\kappa \quad k_1}{\kappa + \frac{k_2 \quad k_1}{2}} < \frac{\kappa \quad k_1}{\kappa} < \frac{\kappa \quad k_1}{\kappa \quad \frac{1}{2}} < \frac{\kappa \quad k_1 + 1}{\kappa + \frac{1}{2}}.$$

Hence, the optimal strategies of the informed traders at $t = 2$ are given in Table C.5.

| Condition | Optimal Strategies of Informed Traders at t = 2 | | |
|---|---|---|---|
| | **State of the Book** | **IH** | **IL** |
| Case $I_1$ $\theta_2^I \quad \frac{\kappa \quad k_2}{\kappa \quad \frac{k_2-k_1}{2}}$ | $(A_1^1, B_1^1)$ | $BX$ | $SX$ |
| | $(A_1^2, B_1^1)$ | $BMO$ | $SMO$ |
| | $(A_1^1, B_1^1+\tau)$ | $BMO$ | $SMO$ |
| | $(A_1^1, B_1^2)$ | $BMO$ | $SMO$ |
| | $(A_1^1 \quad \tau, B_1^1)$ | $BMO$ | $SMO$ |
| Case $I_2$ $\frac{\kappa \quad k_2}{\kappa \quad \frac{k_2-k_1}{2}} < \theta_2^I \quad \frac{\kappa \quad k_1}{\kappa+\frac{k_2-k_1}{2}}$ | $(A_1^1, B_1^1)$ | $BX$ | $SX$ |
| | $(A_1^2, B_1^1)$ | $BDO$ | $SMO$ |
| | $(A_1^1, B_1^1+\tau)$ | $BMO$ | $SMO$ |
| | $(A_1^1, B_1^2)$ | $BMO$ | $SDO$ |
| | $(A_1^1 \quad \tau, B_1^1)$ | $BMO$ | $SMO$ |
| Case $I_3$ $\frac{\kappa \quad k_1}{\kappa+\frac{k_2-k_1}{2}} < \theta_2^I \quad \frac{\kappa \quad k_1}{\kappa}$ | $(A_1^1, B_1^1)$ | $BX$ | $SX$ |
| | $(A_1^2, B_1^1)$ | $BDO$ | $SDO$ |
| | $(A_1^1, B_1^1+\tau)$ | $BMO$ | $SMO$ |
| | $(A_1^1, B_1^2)$ | $BDO$ | $SDO$ |
| | $(A_1^1 \quad \tau, B_1^1)$ | $BMO$ | $SMO$ |
| Case $I_4$ $\frac{\kappa \quad k_1}{\kappa} < \theta_2^I \quad \frac{\kappa \quad k_1}{\kappa \quad \frac{1}{2}}$ | $(A_1^1, B_1^1)$ | $BY$ | $SY$ |
| | $(A_1^2, B_1^1)$ | $BDO$ | $SDO$ |
| | $(A_1^1, B_1^1+\tau)$ | $BMO$ | $SMO$ |
| | $(A_1^1, B_1^2)$ | $BDO$ | $SDO$ |
| | $(A_1^1 \quad \tau, B_1^1)$ | $BMO$ | $SMO$ |
| Case $I_5$ $\frac{\kappa \quad k_1}{\kappa \quad \frac{1}{2}} < \theta_2^I \quad \frac{\kappa \quad k_1+1}{\kappa+\frac{1}{2}}$ | $(A_1^1, B_1^1)$ | $BY$ | $SY$ |
| | $(A_1^2, B_1^1)$ | $BDO$ | $SDO$ |
| | $(A_1^1, B_1^1+\tau)$ | $BDO$ | $SMO$ |
| | $(A_1^1, B_1^2)$ | $BDO$ | $SDO$ |
| | $(A_1^1 \quad \tau, B_1^1)$ | $BMO$ | $SDO$ |
| Case $I_6$ $\frac{\kappa \quad k_1+1}{\kappa+\frac{1}{2}} < \theta_2^I$ | $(A_1^1, B_1^1)$ | $BY$ | $SY$ |
| | $(A_1^2, B_1^1)$ | $BDO$ | $SDO$ |
| | $(A_1^1, B_1^1+\tau)$ | $BDO$ | $SDO$ |
| | $(A_1^1, B_1^2)$ | $BDO$ | $SDO$ |
| | $(A_1^1 \quad \tau, B_1^1)$ | $BDO$ | $SDO$ |

Table C.5: Optimal Strategies of Informed Traders at $t = 2$

where we define by

$$BX = \begin{cases} BMO \text{ if } p_{BLO,2}^{IH,B_1=?} \quad \frac{\kappa \quad k_1}{\delta(\kappa+k_1 \quad 1)}, \\ BLO \text{ if } p_{BLO,2}^{IH,B_1=?} > \frac{\kappa \quad k_1}{\delta(\kappa+k_1 \quad 1)}. \end{cases}$$

$$SX = \begin{cases} SMO \text{ if } p_{SLO,2}^{IL,B_1=?} \quad \frac{\kappa \quad k_1}{\delta(\kappa+k_1 \quad 1)}, \\ SLO \text{ if } p_{SLO,2}^{IL,B_1=?} > \frac{\kappa \quad k_1}{\delta(\kappa+k_1 \quad 1)}. \end{cases}$$

and

$$BY = \begin{cases} BDO \text{ if } p^{IH,B_1=?}_{BLO,2} < \frac{\theta^I_2 \kappa}{\delta(\kappa+k_1 \;\; 1)}, \\ BLO \text{ if } p^{IH,B_1=?}_{BLO,2} \qquad \frac{\theta^I_2 \kappa}{\delta(\kappa+k_1 \;\; 1)}. \end{cases}$$

$$SY = \begin{cases} SDO \text{ if } p^{IH,B_1=?}_{BLO,2} < \frac{\theta^I_2 \kappa}{\delta(\kappa+k_1 \;\; 1)}, \\ SLO \text{ if } p^{IH,B_1=?}_{BLO,2} \qquad \frac{\theta^I_2 \kappa}{\delta(\kappa+k_1 \;\; 1)}. \end{cases}$$

■

We next include a definition and a lemma which will be useful to prove Proposition 2.

**Definition C.3** *Let us consider the following cut-off definitions*

$$\begin{aligned}
\widehat{\theta}_{MO \;\; DO} \quad & \frac{\kappa \;\; k_1 \;\; \delta\left(\kappa \;\; k_1+\lambda\frac{(1 \;\; \pi)}{2}I^{US,B_1=?}_{SLO,2} \;\; (k_2 \;\; k_1)\left(\lambda\pi I^{IH,B_1=?}_{BMO,2}+\frac{1 \;\; \lambda}{2}\right)\right)}{\kappa \;\; \delta\left(\kappa \;\; k_1+\lambda\frac{(1 \;\; \pi)}{2}I^{US,B_1=?}_{SLO,2} \;\; (k_2 \;\; k_1)\left(\lambda\pi I^{IH,B_1=?}_{BMO,2}+\frac{1 \;\; \lambda}{2}\right)\right)}, \\
\overline{\theta}_{MO \;\; DO} \quad & \frac{\kappa \;\; k_1 \;\; \delta\left(\kappa \;\; k_1 \;\; (k_2 \;\; k_1)\left(\lambda\pi+\frac{1 \;\; \lambda}{2}\right)\right)}{\kappa \;\; \delta\left(\kappa \;\; k_1 \;\; (k_2 \;\; k_1)\left(\lambda\pi+\frac{1 \;\; \lambda}{2}\right)\right)}, \\
\widehat{\theta}_{LO \;\; DO} \quad & \frac{\delta\frac{1 \;\; \lambda}{2}(\kappa+k_1 \;\; 1) \;\; \delta\left(\kappa \;\; k_1+\lambda\frac{(1 \;\; \pi)}{2}I^{US,B_1=?}_{SLO,2} \;\; (k_2 \;\; k_1)\left(\lambda\pi I^{IH,B_1=?}_{BMO,2}+\frac{1 \;\; \lambda}{2}\right)\right)}{\kappa \;\; \delta\left(\kappa \;\; k_1+\lambda\frac{(1 \;\; \pi)}{2}I^{US,B_1=?}_{SLO,2} \;\; (k_2 \;\; k_1)\left(\lambda\pi I^{IH,B_1=?}_{BMO,2}+\frac{1 \;\; \lambda}{2}\right)\right)}, \\
\underline{\theta}_{LO \;\; DO} \quad & \frac{\delta\frac{1 \;\; \lambda}{2}(\kappa+k_1 \;\; 1) \;\; \delta\left(\kappa \;\; k_1+\lambda\frac{(1 \;\; \pi)}{2}I^{US,B_1=?}_{SLO,2} \;\; (k_2 \;\; k_1)\frac{1 \;\; \lambda}{2}\right)}{\kappa \;\; \delta\left(\kappa \;\; k_1+\lambda\frac{(1 \;\; \pi)}{2}I^{US,B_1=?}_{SLO,2} \;\; (k_2 \;\; k_1)\frac{1 \;\; \lambda}{2}\right)}, \\
\overline{\theta}_{LO \;\; DO} \quad & \frac{\delta\frac{1 \;\; \lambda}{2}(\kappa+k_1 \;\; 1) \;\; \delta\left(\kappa \;\; k_1 \;\; (k_2 \;\; k_1)\left(\lambda\pi+\frac{1 \;\; \lambda}{2}\right)\right)}{\kappa \;\; \delta\left(\kappa \;\; k_1 \;\; (k_2 \;\; k_1)\left(\lambda\pi+\frac{1 \;\; \lambda}{2}\right)\right)}, \\
\widetilde{\theta}_{LO \;\; DO} \quad & \frac{\delta\frac{1 \;\; \lambda}{2}(\kappa+k_1 \;\; 1) \;\; \delta\left(\kappa \;\; k_1 \;\; (k_2 \;\; k_1)\frac{1 \;\; \lambda}{2}\right)}{\kappa \;\; \delta\left(\kappa \;\; k_1 \;\; (k_2 \;\; k_1)\frac{1 \;\; \lambda}{2}\right)}, \\
\underline{\theta} \quad & \frac{\kappa \;\; k_1}{\kappa}, \text{ and} \\
\overline{\theta} \quad & \frac{\kappa \;\; k_1+1}{\kappa+\frac{1}{2}}.
\end{aligned} \tag{C.2}$$

The cutoffs defined in (C.2) satisfy the following relationships:

$$\widehat{\theta}_{MO \;\; DO} \qquad \overline{\theta}_{MO \;\; DO} < \underline{\theta} \text{ and} \tag{C.3}$$

$$\underline{\theta}_{LO \;\; DO} \qquad \min\left\{\widehat{\theta}_{LO \;\; DO}, \widetilde{\theta}_{LO \;\; DO}\right\} \qquad \max\left\{\widehat{\theta}_{LO \;\; DO}, \widetilde{\theta}_{LO \;\; DO}\right\} \qquad \overline{\theta}_{LO \;\; DO}. \tag{C.4}$$

**Lemma C.1** *Case A. Suppose $k_1 > 1$. Then, a PBE of the game is as follows:*

- $E^D_1$: *$(BMO, SMO, BLO, SLO)$ is the optimal strategy profile at $t = 1$ if*

$$\kappa^I_{MO \;\; LO}\tau \qquad \sigma,\ PIN < \psi^U_{LO \;\; NT} \text{ and } \theta^I_1 \qquad \widehat{\theta}_{MO \;\; DO}.$$

*The beliefs of an uninformed trader at $t = 2$ are: $X^{1,D} = \dfrac{\lambda\pi}{1 \;\; \lambda+\lambda\pi}$, $Y^{1,D} = 0$ and $Z^{1,D} = z \; 2 \; [0,1]$. The optimal choice of an uninformed and an informed trader $t = 2$ are described in Table C.7 and a subset of Table C.5, respectively.*[31]

[31] In the proof of the lemma, we describe for each equilibrium the relevant subset of Table C.5.

- $\mathsf{E}_2^D$: $(BMO, SMO, NT, NT)$ *is the optimal strategy profile at* $t = 1$ *if*

$$\kappa^I_{MO\ \ LO}\tau \quad \sigma,\ PIN \quad \psi^U_{LO\ \ NT},\ \textit{and}\ \theta^I_1 \quad \overline{\theta}_{MO\ \ DO}.$$

*The beliefs of an uninformed trader at* $t = 2$ *are:* $X^{2,D} = \dfrac{\lambda\pi}{1 \quad \lambda + \lambda\pi}$, $Y^{2,D} = p \in [0,1]$ *and* $Z^{2,D} = z \in [0,1]$. *The optimal choice of an uninformed and an informed trader at* $t = 2$ *are described in Table C.8 and a subset of Table C.5, respectively.*

- $\mathsf{E}_3^D$: $(BLO, SLO, BLO, SLO)$ *is the optimal strategy profile at* $t = 1$ *if*

$$\begin{aligned} & \sigma < \kappa^I_{MO\ \ LO}\tau,\ PIN < \psi^U_{LO\ \ NT},\ \textit{and}\ \theta^I_1 \quad \min\{\underline{\theta}, \widehat{\theta}_{LO\ \ DO}\}, \\ \textit{or}\quad & PIN < \psi^U_{LO\ \ NT}\ \textit{and}\ \underline{\theta} < \theta^I_1 \quad \min\{\overline{\theta}, \underline{\theta}_{LO\ \ DO}\}, \\ \textit{or}\quad & \overline{\theta} < \theta^I_1 \quad \underline{\theta}_{LO\ \ DO}. \end{aligned}$$

*The beliefs of an uninformed trader at* $t = 2$ *are:* $X^{3,D} = 0$, $Y^{3,D} = \pi$ *and* $Z^{3,D} = z \in [0,1]$. *The optimal choice of an uninformed and an informed trader* $t = 2$ *are described in Table C.9 and a subset of Table C.5, respectively.*

- $\mathsf{E}_4^D$: $(BLO, SLO, NT, NT)$ *is the optimal strategy profile of a trader at* $t = 1$ *if*

$$\begin{aligned} & \sigma < \kappa^I_{MO\ \ LO}\tau,\ PIN \quad \psi^U_{LO\ \ NT},\ \textit{and}\ \theta^I_1 \quad \min\{\underline{\theta}, \overline{\theta}_{LO\ \ DO}\}, \\ \textit{or}\quad & PIN \quad \psi^U_{LO\ \ NT}\ \textit{and}\ \underline{\theta} < \theta^I_1 \quad \min\{\overline{\theta}, \widetilde{\theta}_{LO\ \ DO}\}. \end{aligned}$$

*The beliefs of an uninformed trader at* $t = 2$ *are:* $X^{4,D} = 0$, $Y^{4,D} = 1$ *and* $Z^{4,D} = z \in [0,1]$. *The optimal choice of an uninformed and an informed trader at* $t = 2$ *are described in Table C.10 and a subset of Table C.5, respectively.*

- $\mathsf{E}_5^D$: $(BDO, SDO, BLO, SLO)$ *is the optimal strategy profile of a trader at* $t = 1$ *if*

$$\begin{aligned} & PIN < \psi^U_{LO\ \ NT},\ \theta^I_1 > \max\{\overline{\theta}_{MO\ \ DO}, \overline{\theta}_{LO\ \ DO}\},\ \textit{and}\ \theta^I_2 \quad \underline{\theta}, \\ \textit{or}\quad & PIN < \psi^U_{LO\ \ NT},\ \widetilde{\theta}_{LO\ \ DO} < \theta^I_1,\ \textit{and}\ \underline{\theta} < \theta^I_2 \quad \overline{\theta}, \\ \textit{or}\quad & \widetilde{\theta}_{LO\ \ DO} < \theta^I_1,\ \textit{and}\ \overline{\theta} < \theta^I_2. \end{aligned}$$

*The beliefs of an uninformed trader at* $t = 2$ *are:* $X^{5,D} = 0$, $Y^{5,D} = 0$ *and* $Z^{5,D} = 1$. *The optimal choice of an uninformed and an informed trader* $t = 2$ *are described in Table C.11 and a subset of Table C.5, respectively.*

- $\mathsf{E}_6^D$: $(BDO, SDO, NT, NT)$ *is the optimal strategy profile of a trader at* $t = 1$ *if*

$$\begin{aligned} & PIN \quad \psi^U_{LO\ \ NT},\ \theta^I_1 > \max\{\overline{\theta}_{MO\ \ DO}, \overline{\theta}_{LO\ \ DO}\},\ \textit{and}\ \theta^I_2 \quad \underline{\theta}, \\ \textit{or}\quad & PIN \quad \psi^U_{LO\ \ NT},\ \widetilde{\theta}_{LO\ \ DO} < \theta^I_1,\ \textit{and}\ \underline{\theta} < \theta^I_2 \quad \overline{\theta}. \end{aligned}$$

*The beliefs of an uninformed trader at* $t = 2$ *are:* $X^{6,D} = 0$, $Y^{6,D} = p \in [0,1]$ *and* $Z^{6,D} = 1$. *The optimal choice of an uninformed and an informed trader at* $t = 2$ *are described in Table C.12 and a subset of Table C.5, respectively.*

Case B. *Suppose* $k_1 = 1$. *Then,* $(BMO, SMO, NT, NT)$ *is the optimal strategy profile at* $t = 1$ *if* $\theta^I_1 \quad \overline{\theta}_{MO\ \ DO}$, *and* $(BDO, SDO, NT, NT)$ *is the optimal strategy profile at* $t = 1$ *if*

$$\begin{aligned} & \theta^I_1 > \max\{\overline{\theta}_{MO\ \ DO}, \overline{\theta}_{LO\ \ DO}\}\ \textit{and}\ \theta^I_2 \quad \min\{\underline{\theta}, \theta^I_1\}, \\ \textit{or}\quad & \widetilde{\theta}_{LO\ \ DO} < \theta^I_1\ \textit{and}\ \underline{\theta} < \theta^I_2 \quad \theta^I_1. \end{aligned}$$

Remark C.1 *Recall that in a Perfect Bayesian Equilibrium beliefs must satisfy Bayes' rule, whenever possible. This occurs along the equilibrium path, not off-the-equilibrium path, where beliefs are indeterminate. This indeterminacy might result in multiplicity of equilibria in sequential games with imperfect information. Note that this may occur in our case when the uninformed trader's beliefs at* $t = 2$ *(i.e.,* $X$*,* $Y$*, or* $Z$*) are indeterminate.*

**Proof of Lemma C.1.** Because of the symmetry of the model, without any loss of generality, we focus on buyers. We present the full proof for one of the possible strategy profile at $t = 1$ that yields an equilibrium. The proofs of all the other 5 equilibria can be found in the Internet Appendix II. Note that in all equilibria the optimal responses of informed traders at $t = 2$ are given in Table C.5. However, in some equilibria not all the 6 cases $I_1 \quad I_6$ are possible and also not all of the 5 states of the book are possible. As a result only a subset of Table C.5 will apply.

$E_1^D$: $(BMO, SMO, BLO, SLO)$

**First step.** In this case $\Omega_0 = 0$, $\Omega_1 = 1$, $\Omega_2 = 0$, $\Omega_3 = 0$, $\Gamma_0 = 0$, $\Gamma_1 = 0$, $\Gamma_2 = 1$, and $\Gamma_3 = 0$. Moreover, $\theta_2^I = \theta_1^I$ and $\theta_2^U = \theta_1^U$. We define by $P \quad p_{BLO,2}^{UB,B_1=?} = p_{SLO,2}^{US,B_1=?}$.

**Second step.** Using Bayes' rule

$$X^{1,D} = \frac{\lambda\pi}{1 \quad \lambda + \lambda\pi},\ Y^{1,D} = 0,\ Z^{1,D} = z\ 2\ [0,1],$$
$$p_{BLO,2}^{UB,B_1=?} = p_{SLO,2}^{US,B_1=?}\ 2\ [0,1],\ \text{and}\ p_{BLO,2}^{IH,B_1=?} = p_{SLO,2}^{IL,B_1=?}\ 2\ [0,1].$$

**Third step.** Using step 2 and taking into account that $p_{BLO,2}^{UB}(B_1 = ?) = p_{SLO,2}^{US}(B_1 = ?)\ 2\ [0,1]$, at $t = 2$ the expected profits of uninformed buyers are as given by Table C.2. Using the symmetry of buyers and sellers, we obtain that the optimal strategy for the uninformed are:

| **Optimal Strategies of Uninformed Traders at $t = 2$** | | |
|---|---|---|
| **State of the Book** | **UB** | **US** |
| $(A_1^1, B_1^1)$ | $NT$ if $P = 0$ or $Z^{1,D}\kappa \quad k_1 \quad 1$<br>$BLO$ if $P > 0$ and $Z^{1,D}\kappa < k_1 \quad 1$ | $NT$ if $P = 0$ or $Z^{1,D}\kappa \quad k_1 \quad 1$<br>$SLO$ if $P > 0$ and $Z^{1,D}\kappa < k_1 \quad 1$ |
| $(A_1^2, B_1^1)$ | $NT$ if $X^{1,D}\kappa \quad \frac{k_2 \quad k_1}{2}$<br>$BDO$ if $\frac{k_2 \quad k_1}{2} < X^{1,D}\kappa \quad k_2$<br>$BDO$ if $k_2 < X^{1,D}\kappa$ and $\theta_2^U > \theta_{X^{1,D}}$<br>$BMO$ if $k_2 < X^{1,D}\kappa$ and $\theta_2^U \quad \theta_{X^{1,D}}$ | $SDO$ if $X^{1,D}\kappa < \frac{k_2 \quad k_1}{2}$<br>$NT$ if $\frac{k_2 \quad k_1}{2} \quad X^{1,D}\kappa$ |
| $(A_1^1, B_1^1 + \tau)$ | $NT$ | $SDO$ |
| $(A_1^1, B_1^2)$ | $BDO$ if $X^{1,D}\kappa < \frac{k_2 \quad k_1}{2}$<br>$NT$ if $\frac{k_2 \quad k_1}{2} \quad X^{1,D}\kappa$ | $NT$ if $X^{1,D}\kappa \quad \frac{k_2 \quad k_1}{2}$<br>$SDO$ if $\frac{k_2 \quad k_1}{2} < X^{1,D}\kappa \quad k_2$<br>$SDO$ if $k_2 < X^{1,D}\kappa$ and $\theta_2^U > \theta_{X^{1,D}}$<br>$SMO$ if $k_2 < X^{1,D}\kappa$ and $\theta_2^U \quad \theta_{X^{1,D}}$ |
| $(A_1^1 \quad \tau, B_1^1)$ | $BDO$ | $NT$ |

Table C.6: Optimal strategies of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BMO, SMO, BLO, SLO)$.

Concerning the informed buyers their expected profits are as given by Table C.1 and the optimal strategy for an informed trader at $t = 2$ is given by Table C.5.

**Fourth step.** Given the optimal response of traders at $t = 2$, we find the optimal action for the traders at $t = 1$ in each of the 6 cases. However, given the nature of this particular equilibrium, we can group cases and analyze them in the following way:

Case $I_1 + I_2 + I_3 : \theta_2^I \quad \frac{\kappa \quad k_1}{\kappa}$

- *Informed traders*

As $\theta_2^I \quad \frac{\kappa \quad k_1}{\kappa}$, informed traders at $t = 1$ have no incentives to deviate from the prescribed strategy profile whenever

$$\kappa \quad k_1 \qquad \frac{1 \quad \lambda}{2}\delta\left(\kappa + k_1 \quad 1\right) \text{ and} \tag{C.5}$$
$$\kappa \quad k_1 \qquad \theta_1^I \kappa + (1 \quad \theta_1^I)\delta\left(\kappa \quad k_1 + \lambda \frac{(1 \quad \pi)}{2} I_{SLO,2}^{US,B_1=?} \quad (k_2 \quad k_1)\left(\lambda\pi I_{BMO,2}^{IH,B_1=?} + \frac{1 \quad \lambda}{2}\right)\right).$$

- *Uninformed traders*

  As $\theta_2^I \quad \frac{\kappa \quad k_1}{\kappa} \quad \frac{\kappa \quad k_1 + 1}{\kappa + \frac{1}{2}}$, uninformed traders at $t = 1$ have no incentives to deviate from the prescribed strategy profile whenever

$$(\lambda\pi + 1 \quad \lambda)(k_1 \quad 1) \quad \lambda\pi\kappa > 0. \tag{C.6}$$

Case $I_4 + I_5 + I_6 : \frac{\kappa \quad k_1}{\kappa} < \theta_2^I$

- *Informed traders*

Consider an informed buyer at $t = 1$. If he chooses a $BMO$, then he obtains

$$\mathsf{E}\left(\Pi_{BMO,1}^{IH}\right) = (\kappa \quad k_1)\,\tau.$$

If instead he deviates towards a $BDO$, he will obtain

$$\mathsf{E}\left(\Pi_{BDO,1}^{IH}\right) = \theta_1^I \kappa\tau + (1 \quad \theta_1^I)\delta\left[\lambda\frac{(1 \quad \pi)}{2} I_{SLO,2}^{US,B_1=?} + (\kappa \quad k_1) \quad (k_2 \quad k_1)\left(\lambda\pi I_{BMO,2}^{IH,B_1=?} + \frac{1 \quad \lambda}{2}\right)\right]\tau.$$

Combining the previous expression and the fact that $\frac{\kappa \quad k_1}{\kappa} < \theta_2^I = \theta_1^I$, it follows that

$$\mathsf{E}\left(\Pi_{BDO,1}^{IH}\right) > \mathsf{E}\left(\Pi_{BMO,1}^{IH}\right) \tag{C.7}$$

is always satisfied, and we conclude that in this case there is no equilibrium in which the strategy profile chosen at $t = 1$ is $(BMO, SMO, BLO, SLO)$.

**Fifth step.** Based on the above, nobody at $t = 1$ has unilateral incentives to deviate whenever $\theta_1^I \quad \frac{\kappa \quad k_1}{\kappa}$ and (C.5), (C.6) and (C.7) are satisfied. These conditions can be rewritten as

$$\kappa_{MO \quad LO}^I \tau \quad \sigma, \; PIN < \psi_{LO \quad NT}^U \text{ and } \theta_1^I \quad \widehat{\theta}_{MO \quad DO}. \tag{C.8}$$

Finally, in the following tables we include the moves that are in the equilibrium path taking into account the conditions that must be satisfied if $(BMO, SMO, BLO, SLO)$ is the strategy profile chosen at $t = 1$ and the fact that in this case $\theta_2^I = \theta_1^I$.

Concerning uninformed traders notice that the condition $(\lambda\pi + 1 \quad \lambda)(k_1 \quad 1) \quad \lambda\pi\kappa > 0$ implies that $X^{1,D}\kappa < k_1 \quad 1 < k_2$. Hence, the optimal choice of uninformed traders at $t = 2$ is as follows:

| Condition | Optimal Choice of Uninformed Traders at t = 2 | | |
|---|---|---|---|
| | **State of the Book** | **UB** | **US** |
| Case $U_1^{E_1^D}$<br>$k_1 \quad 1 \quad \frac{k_2 \quad k_1}{2}$<br>or<br>$k_1 \quad 1 > \frac{k_2 \quad k_1}{2}$<br>and $X^{1,D}\kappa < \frac{k_2 \quad k_1}{2}$ | $(A_1^2, B_1^1)$<br>$(A_1^1, B_1^1 + \tau)$<br>$(A_1^1, B_1^2)$<br>$(A_1^1 \quad \tau, B_1^1)$ | $NT$<br>$NT$<br>$BDO$<br>$BDO$ | $SDO$<br>$SDO$<br>$NT$<br>$NT$ |
| Case $U_2^{E_1^D}$<br>$k_1 \quad 1 > \frac{k_2 \quad k_1}{2}$<br>and $X^{1,D}\kappa = \frac{k_2 \quad k_1}{2}$ | $(A_1^2, B_1^1)$<br>$(A_1^1, B_1^1 + \tau)$<br>$(A_1^1, B_1^2)$<br>$(A_1^1 \quad \tau, B_1^1)$ | $NT$<br>$NT$<br>$NT$<br>$BDO$ | $NT$<br>$SDO$<br>$NT$<br>$NT$ |
| Case $U_3^{E_1^D}$<br>$k_1 \quad 1 > \frac{k_2 \quad k_1}{2}$<br>and $\frac{k_2 \quad k_1}{2} < X^{1,D}\kappa < k_1 \quad 1$ | $(A_1^2, B_1^1)$<br>$(A_1^1, B_1^1 + \tau)$<br>$(A_1^1, B_1^2)$<br>$(A_1^1 \quad \tau, B_1^1)$ | $BDO$<br>$NT$<br>$NT$<br>$BDO$ | $NT$<br>$SDO$<br>$SDO$<br>$NT$ |

Table C.7: Optimal choice of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BMO, SMO, BLO, SLO)$

In relation to informed traders the optimal choice at $t = 2$ is obtained by selecting in Table C.5 the cases $I_1, I_2$ and $I_3$ and the following possible prices $(A_1^2, B_1^1)$, $(A_1^1, B_1^1 + \tau)$, $(A_1^1, B_1^2)$, $(A_1^1 \quad \tau, B_1^1)$.

$E_2^D$: $(BMO, SMO, NT, NT)$

In this case the beliefs of an uninformed trader at $t = 2$ are: $X^{2,D} = \dfrac{\lambda\pi}{1 \quad \lambda + \lambda\pi}$, $Y^{2,D} = p$ 2 $[0,1]$ and $Z^{2,D} = z$ 2 $[0,1]$. In addition, nobody at $t = 1$ has unilateral incentives to deviate whenever

$$\kappa^I_{MO \quad LO}\tau \quad \sigma,\ PIN \quad \psi^U_{LO \quad NT}, \text{ and } \theta^I_1 \quad \overline{\theta}_{MO \quad DO}. \tag{C.9}$$

Finally, we include the decisions that are in the equilibrium path and that in this case $\theta^I_2 = \theta^I_1$. Furthermore, in relation to uninformed traders, and taking into account that a necessary condition for this equilibrium tells us that $k_1 \quad 1 \quad X^{2,D}\kappa$, the following cases can be distinguished

| Condition | Optimal Choice of Uninformed Traders at $\mathbf{t = 2}$ | | |
|---|---|---|---|
| | **State of the Book** | **UB** | **US** |
| Case $U_1^{E_2^D}$<br>$k_1 \quad 1 \quad X^{2,D}\kappa < \frac{k_2 \quad k_1}{2}$ | $(A_1^1, B_1^1)$<br>$(A_1^2, B_1^1)$<br>$(A_1^1, B_1^2)$ | $NT$<br>$NT$<br>$BDO$ | $NT$<br>$SDO$<br>$NT$ |
| Case $U_2^{E_2^D}$<br>$k_1 \quad 1 < X^{2,D}\kappa = \frac{k_2 \quad k_1}{2}$ | $(A_1^1, B_1^1)$<br>$(A_1^2, B_1^1)$<br>$(A_1^1, B_1^2)$ | $NT$<br>$NT$<br>$NT$ | $NT$<br>$NT$<br>$NT$ |
| Case $U_3^{E_2^D}$<br>$\max\left\{k_1 \quad 1, \frac{k_2 \quad k_1}{2}\right\} < X^{2,D}\kappa \quad k_2$<br>or<br>$k_2 < X^{2,D}\kappa$ and $\theta_2^U > \theta_{X^{2,D}}$ | $(A_1^1, B_1^1)$<br>$(A_1^2, B_1^1)$<br>$(A_1^1, B_1^2)$ | $NT$<br>$BDO$<br>$NT$ | $NT$<br>$NT$<br>$SDO$ |
| Case $U_4^{E_2^D}$<br>$k_2 < X^{2,D}\kappa$ and $\theta_2^U \quad \theta_{X^{2,D}}$ | $(A_1^1, B_1^1)$<br>$(A_1^2, B_1^1)$<br>$(A_1^1, B_1^2)$ | $NT$<br>$BMO$<br>$NT$ | $NT$<br>$NT$<br>$SMO$ |

Table C.8: Optimal choice of uniformed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BMO, SMO, NT, NT)$

In relation to informed traders the optimal choice at $t = 2$ can be obtained by selecting in Table C.5 only the cases $I_1$, $I_2$ and $I_3$, and the following possible prices: $(A_1^1, B_1^1), (A_1^2, B_1^1)$, and $(A_1^1, B_1^2)$, with $BX = BMO$, $SX = SMO$, $BY = BDO$, and $SY = SDO$.

$E_3^D$: $(BLO, SLO, BLO, SLO)$

In this case the beliefs of an uninformed trader at $t = 2$ are: $X^{3,D} = 0$, $Y^{3,D} = \pi$ and $Z^{3,D} = z \in [0, 1]$. In addition, the conditions under which nobody is willing to deviate at $t = 1$ are

$$
\begin{aligned}
& \sigma < \kappa^I_{MO \quad LO}\tau,\ PIN < \psi^U_{LO \quad NT}, \text{ and } \theta_1^I \quad \min\{\underline{\theta}, \widehat{\theta}_{LO \quad DO}\}, \\
\text{or} \quad & PIN < \psi^U_{LO \quad NT} \text{ and } \underline{\theta} < \theta_1^I \quad \min\{\overline{\theta}, \underline{\theta}_{LO \quad DO}\}, \\
\text{or} \quad & k_1 > 1 \text{ and } \overline{\theta} < \theta_1^I \quad \underline{\theta}_{LO \quad DO}.
\end{aligned}
\tag{C.10}
$$

Finally, we include the moves that are in the equilibrium path taking into account the conditions that must be satisfied if $(BLO, SLO, BLO, SLO)$ is the strategy profile chosen at $t = 1$ and $\theta_2^I = \theta_1^I$.

Concerning uninformed traders, it follows that their optimal choices at $t = 2$ are

| Condition | Optimal Choice of Uninformed Traders at t = 2 | | |
|---|---|---|---|
| | **State of the Book** | **UB** | **US** |
| Case $U_1^{E_3^D}$<br>$Y^{3,D}\kappa \leq \frac{1}{2}$ | $(A_1^2, B_1^1)$<br>$(A_1^1, B_1^1+\tau)$<br>$(A_1^1, B_1^2)$<br>$(A_1^1-\tau, B_1^1)$ | $NT$<br>$NT$<br>$BDO$<br>$BDO$ | $SDO$<br>$SDO$<br>$NT$<br>$NT$ |
| Case $U_2^{E_3^D}$<br>$\frac{1}{2} < Y^{3,D}\kappa \leq k_1$<br>or<br>$Y^{3,D}\kappa > k_1$and $\theta_2^U > \theta_{Y^{3,D}}$ | $(A_1^2, B_1^1)$<br>$(A_1^1, B_1^1+\tau)$<br>$(A_1^1, B_1^2)$<br>$(A_1^1-\tau, B_1^1)$ | $NT$<br>$BDO$<br>$BDO$<br>$NT$ | $SDO$<br>$NT$<br>$NT$<br>$SDO$ |
| Case $U_3^{E_3^D}$<br>$Y^{3,D}\kappa > k_1$ and $\theta_2^U \leq \theta_{Y^{3,D}}$ | $(A_1^2, B_1^1)$<br>$(A_1^1, B_1^1+\tau)$<br>$(A_1^1, B_1^2)$<br>$(A_1^1-\tau, B_1^1)$ | $NT$<br>$BMO$<br>$BDO$<br>$NT$ | $SDO$<br>$NT$<br>$NT$<br>$SMO$ |

Table C.9: Optimal choice of uninformed traders when the strategy profile at $t = 1$ is $(BLO, SLO, BLO, SLO)$.

In relation to informed traders the optimal choice at $t = 2$ is obtained by selecting in Table C.5 all the cases $I_1 - I_6$ and the following possible prices: $(A_1^2, B_1^1)$, $(A_1^1, B_1^1+\tau)$, $(A_1^1, B_1^2)$, and $(A_1^1-\tau, B_1^1)$.

$E_4^D$: $(BLO, SLO, NT, NT)$

In this case the beliefs of an uninformed trader at $t = 2$ are: $X^{4,D} = 0$, $Y^{4,D} = 1$ and $Z^{4,D} = z \in [0, 1]$. In addition, the conditions under which nobody is willing to deviate at $t = 1$ are

$$\begin{array}{rl} & \sigma < \kappa^I_{MO-LO}\tau,\ PIN \geq \psi^U_{LO-NT}, \text{ and } \theta_1^I \geq \min\{\underline{\theta}, \overline{\theta}_{LO-DO}\}, \\ \text{or} & PIN \geq \psi^U_{LO-NT} \text{ and } \underline{\theta} < \theta_1^I \leq \min\{\overline{\theta}, \widetilde{\theta}_{LO-DO}\}. \end{array} \quad \text{(C.11)}$$

Finally, we include the moves that are in the equilibrium path taking into account the conditions that must be satisfied if $(BLO, SLO, NT, NT)$ is the strategy profile chosen at $t = 1$ and the fact that in this case $\theta_2^I = \theta_1^I$ and $\theta_2^U = \theta_1^U$. Concerning the uninformed traders, we have

| Condition | Optimal Choice of Uninformed Traders at t = 2 | | |
|---|---|---|---|
| | **State of the Book** | **UB** | **US** |
| Case $U_1^{E_4^D}$<br>$\theta_2^U > \theta_{Y^{4,D}}$ | $(A_1^1, B_1^1)$<br>$(A_1^2, B_1^1)$<br>$(A_1^1, B_1^1+\tau)$<br>$(A_1^1, B_1^2)$<br>$(A_1^1-\tau, B_1^1)$ | $NT$<br>$NT$<br>$BMO$<br>$BDO$<br>$NT$ | $NT$<br>$SDO$<br>$NT$<br>$NT$<br>$SMO$ |
| Case $U_2^{E_4^D}$<br>$\theta_2^U \leq \theta_{Y^{4,D}}$ | $(A_1^1, B_1^1)$<br>$(A_1^2, B_1^1)$<br>$(A_1^1, B_1^1+\tau)$<br>$(A_1^1, B_1^2)$<br>$(A_1^1-\tau, B_1^1)$ | $NT$<br>$NT$<br>$BDO$<br>$BDO$<br>$NT$ | $NT$<br>$SDO$<br>$NT$<br>$NT$<br>$SDO$ |

Table C.10: Optimal choice of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BLO, SLO, NT, NT)$.

In relation to informed traders the optimal choice at $t = 2$ is obtained by selecting in Table C.5 all the cases $I_1 - I_5$, for all the possible pairs of best prices, with $BX = BMO$, $SX = SMO$, $BY = BDO$, and $SY = SDO$.

$E_5^D$: $(BDO, SDO, BLO, SLO)$
In this case the beliefs of an uninformed trader at $t = 2$ are: $X^{5,D} = 0$, $Y^{5,D} = 0$ and $Z^{5,D} = 1$. In addition, nobody at $t = 1$ has unilateral incentives to deviate from $(BDO, SDO, BLO, SLO)$ whenever

$$\begin{array}{ll} & PIN < \psi^U_{LO-NT},\ \theta_1^I > \max\{\theta_{MO-DO}, \overline{\theta}_{LO-DO}\}, \text{ and } \theta_2^I \leq \underline{\theta}, \\ \text{or} & PIN < \psi^U_{LO-NT},\ \widetilde{\theta}_{LO-DO} < \theta_1^I, \text{ and } \underline{\theta} < \theta_2^I \leq \overline{\theta}, \\ \text{or} & k_1 > 1,\ \widetilde{\theta}_{LO-DO} < \theta_1^I, \text{ and } \overline{\theta} < \theta_2^I. \end{array} \tag{C.12}$$

Notice that in this equilibrium we always have $\theta_2^I < \theta_1^I$. Furthermore, the optimal responses of uninformed traders are

| State of the book | UB | US |
|---|---|---|
| $(A_1^1, B_1^1)$ | $NT$ | $NT$ |
| $(A_1^2, B_1^1)$ | $NT$ | $SDO$ |
| $(A_1^1, B_1^1 + \tau)$ | $NT$ | $SDO$ |
| $(A_1^1, B_1^2)$ | $BDO$ | $NT$ |
| $(A_1^1 - \tau, B_1^1)$ | $BDO$ | $NT$ |

Table C.11: Optimal responses of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BDO, SDO, BLO, SLO)$.

In relation to informed traders the optimal choice at $t = 2$ is obtained by selecting in Table C.5 all the cases $I_1 - I_6$, for all the possible pairs of best prices, with $BX = BMO$, $SX = SMO$, $BY = BDO$, and $SY = SDO$.

$E_6^D$: $(BDO, SDO, NT, NT)$
In this case the beliefs of an uninformed trader at $t = 2$ are: $X^{6,D} = 0$, $Y^{6,D} = p \in [0, 1]$ and $Z^{6,D} = 1$. In addition, nobody at $t = 1$ has unilateral incentives to deviate from $(BDO, SDO, NT, NT)$ whenever

$$\begin{array}{ll} & PIN \geq \psi^U_{LO-NT},\ \theta_1^I > \max\{\theta_{MO-DO}, \overline{\theta}_{LO-DO}\}, \text{ and } \theta_2^I \leq \underline{\theta}, \\ \text{or} & PIN \geq \psi^U_{LO-NT},\ \widetilde{\theta}_{LO-DO} < \theta_1^I, \text{ and } \underline{\theta} < \theta_2^I \leq \overline{\theta}, \\ \text{or} & k_1 = 1,\ \widetilde{\theta}_{LO-DO} < \theta_1^I \text{ and } \overline{\theta} < \theta_2^I \leq \theta_1^I. \end{array} \tag{C.13}$$

Notice that in this equilibrium we also have that $\theta_2^I \leq \theta_1^I$. Furthermore, the optimal responses of uninformed traders are

| Optimal Choice of Uninformed Traders at $t = 2$ | | |
|---|---|---|
| **State of the Book** | **UB** | **US** |
| $(A_1^1, B_1^1)$ | $NT$ | $NT$ |
| $(A_1^2, B_1^1)$ | $NT$ | $SDO$ |
| $(A_1^1, B_1^2)$ | $BDO$ | $NT$ |

Table C.12: Optimal choice of uninformed traders at $t = 2$ when the strategy profile at $t = 1$ is $(BDO, SDO, NT, NT)$.

In relation to informed traders the optimal choice at $t = 2$ can be obtained by selecting in Table C.5 all the cases $I_1 - I_6$ and the following possible prices: $(A_1^1, B_1^1)$, $(A_1^2, B_1^1)$, $(A_1^1, B_1^2)$, with $BX = BMO$, $SX = SMO$, $BY = BDO$, and $SY = SDO$.

Finally, note that substituting $k_1 = 1$ (Case B) into the expressions of $\kappa^I_{MO-LO}$ and $\psi^U_{LO-NT}$, we have that

$$\kappa^I_{MO-LO} = \frac{1}{1 - \frac{1}{2}\delta(1 - \lambda)} \text{ and } \psi^U_{LO-NT} = 0.$$

Moreover, since $\kappa^I_{MO-LO} < 2$, it follows that $\kappa^I_{MO-LO}\tau < \sigma$ and $PIN \geq \psi^U_{LO-NT}$. Therefore, using (C.8)-(C.13), we have that when $k_1 = 1$, the conditions related to $E_1^D$, $E_3^D$ and $E_5^D$ are not satisfied. Moreover, as in this case the expected profits of a $MO$ are higher than those of a $LO$ for an informed trader, $E_4^D$ cannot be an equilibrium when $k_1 = 1$. Thus, in this case we have two possible equilibria: $E_2^D$ and $E_6^D$. Specifically, $(BMO, SMO, NT, NT)$ is the optimal strategy profile at $t = 1$ if $\theta_1^I \quad \overline{\theta}_{MO \quad DO}$, and $(BDO, SDO, NT, NT)$ is the optimal strategy profile at $t = 1$ if

$$\begin{aligned} &\theta_1^I > \max f\overline{\theta}_{MO \quad DO}, \overline{\theta}_{LO \quad DO} g \text{ and } \theta_2^I \quad \min\{\underline{\theta}, \theta_1^I\}, \\ \text{or} \quad &\widetilde{\theta}_{LO \quad DO} < \theta_1^I \text{ and } \underline{\theta} < \theta_2^I \quad \theta_1^I. \end{aligned}$$

■

**Proof of Proposition 2.** **Case A.** We consider the same four possible cases, depending on the initial conditions of the single-venue market.

**Case A.1:** $\sigma < \kappa_{MO \quad LO}^I \tau$ and $PIN < \psi_{LO \quad NT}^U$

We start with market conditions such that the prevailing equilibrium is $E_3^{ND}$, where conditions $(B.8)$ and $(B.9)$ are satisfied. When there is access to the $DP$, out of the 6 equilibria, there are only two possible equilibria that satisfy these conditions: $E_3^D$ and $E_5^D$. From Lemma C.1 we can see that $E_3^D$ is an equilibrium if conditions $(C.10)$ are satisfied. Using the relationship between the cutoffs in this case (see Internet Appendix II for a full proof), we conclude that $E_3^D$ is an equilibrium whenever

$$\begin{aligned} &\theta_1^I \quad \underline{\theta}_{LO \quad DO} \quad \text{if } \underline{\theta} < \widehat{\theta}_{LO \quad DO}, \\ \text{or} \quad &\theta_1^I \quad \widehat{\theta}_{LO \quad DO} \quad \text{if } \widehat{\theta}_{LO \quad DO} \quad \underline{\theta}. \end{aligned}$$

On the other hand, $E_5^D$ is an equilibrium if conditions $(C.12)$ are satisfied, they can be rewritten as

$$\begin{aligned} &\theta_1^I > \overline{\theta}_{LO \quad DO} \text{ and } \theta_2^I \quad \underline{\theta}, \\ \text{or} \quad &\widetilde{\theta}_{LO \quad DO} < \theta_1^I \text{ and } \underline{\theta} < \theta_2^I. \end{aligned}$$

As a result, when $\sigma < \kappa_{MO \quad LO}^I \tau$ and $PIN < \psi_{LO \quad NT}^U$, the optimal strategy profiles at $t = 1$ are

$$\begin{cases} (BLO, SLO, BLO, SLO) & \text{if } \theta_1^I \quad \theta_{LO \quad LO}^1, \\ (BDO, SDO, BLO, BLO) & \text{if } \theta_1^I > \theta_{DO \quad LO}^1, \end{cases}$$

where

$$\begin{aligned} \theta_{LO \quad LO}^1 &= \begin{cases} \widehat{\theta}_{LO \quad DO} & \text{if } \widehat{\theta}_{LO \quad DO} < \underline{\theta}, \\ \underline{\theta}_{LO \quad DO} & \text{otherwise.} \end{cases} \\ \theta_{DO \quad LO}^1 &= \begin{cases} \overline{\theta}_{LO \quad DO} & \text{if } \theta_2^I \quad \underline{\theta}, \\ \widetilde{\theta}_{LO \quad DO} & \text{otherwise.} \end{cases} \end{aligned} \tag{C.14}$$

**Case A.2:** $\sigma < \kappa_{MO \quad LO}^I \tau$ and $PIN \quad \psi_{LO \quad NT}^U$

We start with market conditions such that the prevailing equilibrium is $E_4^{ND}$, where conditions $(B.10)$ and $(B.11)$ are satisfied. When there is access to the $DP$, out of the 6 equilibria there are only three possible equilibria that satisfy these conditions: $E_4^D$, $E_5^D$, and $E_6^D$. From Lemma C.1 we can see that $E_4^D$ is an equilibrium if conditions $(C.11)$ are satisfied, then

$$\theta_1^I \quad \overline{\theta}_{LO \quad DO}.$$

On the other hand, $E_5^D$ is an equilibrium if conditions $(C.12)$ are satisfied, they can be rewritten as

$$\widetilde{\theta}_{LO \quad DO} < \theta_1^I \text{ and } \overline{\theta} < \theta_2^I.$$

Finally, $E_6^D$ is an equilibrium if conditions $(C.13)$ are satisfied, and they can be rewritten as

$$\begin{aligned} &\theta_1^I > \overline{\theta}_{LO \quad DO} \text{ and } \theta_2^I \quad \underline{\theta}, \\ \text{or} \quad &\widetilde{\theta}_{LO \quad DO} < \theta_1^I \text{ and } \underline{\theta} < \theta_2^I \quad \overline{\theta}. \end{aligned}$$

As a result, the optimal strategy profiles of a trader at $t = 1$ are

$$\begin{cases} (BLO, SLO, NT, NT) & \text{if} \quad \theta_1^I \leq \theta^{22}_{LO \to NT}, \\ (BDO, SDO, NT, NT) & \text{if} \quad \theta^{22}_{DO \to NT} < \theta_1^I \leq \theta^{22}_{DO \to LO}, \\ (BDO, SDO, BLO, BLO) & \text{if} \quad \theta_1^I > \theta^{22}_{DO \to LO}, \end{cases}$$

where

$$\begin{aligned} \theta^{22}_{LO \to NT} &= \begin{cases} \min\{\overline{\theta}, \widetilde{\theta}_{LO \to DO}\} & \text{if} \quad \underline{\theta} < \overline{\theta}_{LO \to DO}, \\ \overline{\theta}_{LO \to DO} & \text{otherwise}, \end{cases} \\ \theta^{22}_{DO \to NT} &= \begin{cases} \overline{\theta}_{LO \to DO} & \text{if} \quad \theta_2^I \leq \underline{\theta}, \\ \widetilde{\theta}_{LO \to DO} & \text{if} \quad \underline{\theta} < \theta_2^I \leq \overline{\theta}, \quad \text{and} \\ 1 & \text{if} \quad \overline{\theta} < \theta_2^I, \end{cases} \\ \theta^{22}_{DO \to LO} &= \begin{cases} 1 & \text{if} \quad \theta_2^I \leq \overline{\theta}, \\ \widetilde{\theta}_{LO \to DO} & \text{if} \quad \text{otherwise}. \end{cases} \end{aligned} \tag{C.15}$$

Case A.3: $\kappa^I_{MO \to LO}\tau \leq \sigma$ and $PIN < \psi^U_{LO \to NT}$

We start with market conditions such that the prevailing equilibrium is $E_1^{ND}$, where conditions $(B.3)$ and $(B.4)$ are satisfied. When there is access to the $DP$, out of the 6 equilibria there are only two possible equilibria that satisfy these conditions: $E_1^D$ and $E_5^D$. From Lemma C.1 we can see that $E_1^D$ is an equilibrium if conditions $(C.8)$ are satisfied. Similarly, $E_5^D$ is an equilibrium if conditions $(C.12)$ are satisfied, they can be rewritten as

$$\begin{aligned} &\theta_1^I > \overline{\theta}_{MO \to DO} \text{ and } \theta_2^I \leq \underline{\theta}, \\ \text{or} \quad &\widetilde{\theta}_{LO \to DO} < \theta_1^I \text{ and } \underline{\theta} < \theta_2^I. \end{aligned}$$

As a result, the optimal strategy profiles of a trader at $t = 1$ are :

$$\begin{cases} (BMO, SMO, BLO, BLO) & \text{if} \quad \theta_1^I \leq \widehat{\theta}_{MO \to DO}, \\ (BDO, SDO, BLO, BLO) & \text{if} \quad \theta_1^I > \theta^{21}_{DO \to LO}, \end{cases}$$

where

$$\theta^{21}_{DO \to LO} = \begin{cases} \overline{\theta}_{MO \to DO} & \text{if} \quad \theta_2^I \leq \underline{\theta}, \\ \widetilde{\theta}_{LO \to DO} & \text{otherwise}. \end{cases} \tag{C.16}$$

Case A.4: $\kappa^I_{MO \to LO}\tau \leq \sigma$ and $PIN \geq \psi^U_{LO \to NT}$

We start with market conditions such that the prevailing equilibrium is $E_2^{ND}$, where conditions $(B.6)$ and $(B.7)$ are satisfied. When there is access to the $DP$, out of the 6 equilibria there are only three possible equilibria that satisfy these conditions: $E_2^D$, $E_5^D$, and $E_6^D$. From Lemma C.1 we can see that $E_2^D$ is an equilibrium if conditions $(C.9)$ are satisfied. Similarly, $E_5^D$ is an equilibrium if conditions $(C.12)$ are satisfied, they can be rewritten as

$$\widetilde{\theta}_{LO \to DO} < \theta_1^I \text{ and } \overline{\theta} < \theta_2^I.$$

Finally, $E_6^D$ is an equilibrium if conditions $(C.13)$ are satisfied and in this case they can be rewritten as

$$\begin{aligned} &\theta_1^I > \overline{\theta}_{MO \to DO} \text{ and } \theta_2^I \leq \underline{\theta}, \\ \text{or} \quad &\widetilde{\theta}_{LO \to DO} < \theta_1^I \text{ and } \underline{\theta} < \theta_2^I \leq \overline{\theta}. \end{aligned}$$

As a result, the optimal strategy profiles of a trader at $t = 1$ are

$$\begin{cases} (BMO, SMO, NT, NT) & \text{if} \quad \theta_1^I \quad \overline{\theta}_{MO \quad DO}, \\ (BDO, SDO, NT, NT) & \text{if} \quad \theta^3_{DO \quad NT} < \theta_1^I \quad \theta^3_{DO \quad LO}, \\ (BDO, SDO, BLO, BLO) & \text{if} \quad \theta_1^I > \theta^3_{DO \quad LO}, \end{cases}$$

where

$$\theta^3_{DO \quad NT} = \begin{cases} \overline{\theta}_{MO \quad DO} & \text{if} \quad \theta_2^I \quad \underline{\theta}, \\ \widetilde{\theta}_{LO \quad DO} & \text{if} \quad \underline{\theta} \le \theta_2^I \quad \overline{\theta}, \\ 1 & \text{if} \quad \overline{\theta} < \theta_2^I. \end{cases} \quad \text{and} \tag{C.17}$$

$$\theta^3_{DO \quad LO} = \begin{cases} 1 & \text{if} \quad \theta_2^I \quad \overline{\theta}, \\ \widetilde{\theta}_{LO \quad DO} & \text{if} \quad \overline{\theta} < \theta_2^I. \end{cases}$$

**Case B.** From Lemma C.1, one can see that when there is access to the $DP$, out of the 6 equilibria there are only two possible equilibria that satisfy these conditions: $E_2^D$ and $E_6^D$. Note that $E_2^D$ is an equilibrium if conditions $(C.9)$ are satisfied, while $E_6^D$ is an equilibrium if

$$\theta_1^I > \overline{\theta}_{MO \quad DO} \text{ and } \theta_2^I \quad \underline{\theta},$$
$$\text{or} \quad \widetilde{\theta}_{LO \quad DO} < \theta_1^I \text{ and } \underline{\theta} < \theta_2^I.$$

As a result, the optimal strategy profiles of a trader at $t = 1$ are

$$\begin{cases} (BMO, SMO, NT, NT) & \text{if} \quad \theta_1^I \quad \overline{\theta}_{MO \quad DO}, \\ (BDO, SDO, NT, NT) & \text{if} \quad \widehat{\theta}^3_{DO \quad NT} < \theta_1^I, \end{cases}$$

where

$$\widehat{\theta}^3_{DO \quad NT} = \begin{cases} \overline{\theta}_{MO \quad DO} & \text{if} \quad \theta_2^I \quad \underline{\theta}, \\ \widetilde{\theta}_{LO \quad DO} & \text{if} \quad \theta_2^I > \underline{\theta}. \end{cases} \tag{C.18}$$

■

**Proof of Propositions 3, 4, 5 and 6.** See Internet Appendix III. ■

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