Online Appendix to:
Silent Financial Interests and Product Innovation

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04 June 2018

APPENDIX A

Proof of Lemma 1. We start solving backwards and use the first order condition at $t = 2$: $\frac{\partial \Pi_i}{\partial p_i} = 0$ for $i = 1, 2$. Hence, the best-replies for firms $i, j = 1, 2$ and $i \neq j$ are

$$p_i(\omega_i; \omega_j) = \frac{1}{2}\left[t + \rho(x_i - x_j) + \frac{(1 + \omega_i - \omega_j)p_j}{(1 - \omega_j)}\right].$$

Solving the system of equations for prices, we get:

$$p_i(\omega_i; \omega_j) = (1 - \omega_i)\,\frac{t\,[3(1 - \omega_j) + \omega_i] + \rho\,(x_i - x_j)\,(1 - \Omega)}{(3 - \Omega)\,(1 - \Omega)}.$$

Substituting these expressions into profits for each firm we can then solve at $t = 1$ for the optimal amount of R&D expenditures: $\frac{\partial \Pi_i}{\partial x_i} = 0$ for $i, j = 1, 2$ with $i \neq j$ such that the second order condition, $r < \lambda\,(3 - \Omega)^2$ is satisfied. The stability condition requires that $\left|\frac{\partial^2 \Pi_i}{\partial x_i^2}\frac{\partial^2 \Pi_j}{\partial x_j^2}\right| > \left|\frac{\partial^2 \Pi_i}{\partial x_i \partial x_j}\frac{\partial^2 \Pi_j}{\partial x_j \partial x_i}\right|$, which is equivalent to $r < \frac{\lambda(3-\Omega)^2}{2}$. From the first order condition, we obtain:

$$x_i(\omega; \omega) = \frac{-\rho t\,(1 - \omega)\,(2\omega - 3)}{\lambda t\,(2\omega - 3)^2 - \rho^2} - \frac{\rho^2\,(1 - \omega)}{(1 - \omega)\left[\lambda t\,(2\omega - 3)^2 - \rho^2\right]}x_j.$$

$$x_i(\omega_i; \omega_j) = \rho\left[\frac{t\,(3 - 5\omega_i + \omega_i\Omega) - \rho x_j}{t\lambda\,(3 - \Omega)^2 - \rho^2}\right].$$

Solving the system of equations for $x_i(\omega_i; \omega_j)$ and $x_j(\omega_i; \omega_j)$, we obtain the optimal amount of R&D for each firm, which can then be substituted to obtain equilibrium prices, and subsequently, equilibrium R&D and market shares. Letting that $\omega_i > \omega_j$, the interior equilibrium requires that $x_i^*(\omega_i; \omega_j) > 0, p_i^*(\omega_i; \omega_j) > 0, s_i^*(\omega_i; \omega_j) > 0$, which is equivalent to $\frac{r}{\lambda} < \frac{(3-5\omega_i+\omega_i\Omega)(3-\Omega)}{(2-\Omega)}$ (note that $\frac{(3-5\omega_i+\omega_i\Omega)(3-\Omega)}{(2-\Omega)} < \frac{(3-\Omega)^2}{2}$). Equilibrium expressions for the special cases can be easily derived from these general formulae. ■

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APPENDIX B

Proof of Proposition 1.

Using Lemma 1 and comparing the equilibrium outcomes between firms, we obtain:

$$x_i^*(\omega_i;\omega_j) - x_j^*(\omega_i;\omega_j) = \frac{t\rho\,(\omega_j-\omega_i)\,(5-\Omega)}{t\lambda\,(3-\Omega)^2-2\rho^2},$$

hence $sign\{x_i^*(\omega_i;\omega_j) - x_j^*(\omega_i;\omega_j)\} = sign\{\omega_j - \omega_i\}$.With regards to prices, we find that:

$$p_i^*(\omega_i;\omega_j) - p_j^*(\omega_i;\omega_j) = t\,(\omega_i-\omega_j)\,\frac{t\lambda(3-\Omega)-\rho^2(4-\Omega)}{t\lambda\,(3-\Omega)^2-2\rho^2},$$

hence $sign\{p_i^*(\omega_i;\omega_j) - p_j^*(\omega_i;\omega_j)\} = sign\{(\omega_i-\omega_j)\left[\frac{\lambda(3-\Omega)}{(4-\Omega)} - r\right]\}$. In terms of market shares:

$$s_i^*(\omega_i;\omega_j) - s_j^*(\omega_i;\omega_j) = \frac{(\omega_j-\omega_i)\left[\rho^2+t\lambda(3-\Omega)\right]}{t\lambda\,(3-\Omega)^2-2\rho^2},$$

hence $sign\{s_i^*(\omega_i;\omega_j) - s_j^*(\omega_i;\omega_j)\} = sign\{\omega_j-\omega_i\}$. The comparison of the special cases follows immediately. ■

APPENDIX C

Proof of Proposition 2.

Using Lemma 1, and letting $\omega_i > \omega_j$, for each firm $i,j=1,2$ such that $i \neq j$, we compare the equilibrium outcomes of each firm with and without PPO. We obtain:

$$x_i^*(\omega_i;\omega_j) - x_i^*(0;0) = \rho\frac{t\lambda\,(3-\Omega)\,[\omega_j\,(6-\omega_j)-\omega_i\,(9-2\omega_i-\omega_j)]+\rho^2\Omega}{3\lambda\,(3-\Omega)\left[t\lambda\,(3-\Omega)^2-2\rho^2\right]},$$

and

$$sign\{x_i^*(\omega_i;\omega_j) - x_i^*(0;0)\} = sign\{r - \lambda\theta_{x,i}(\omega_i,\omega_j)\},$$

where

$$\theta_{x,i} \equiv (3-\Omega)\,\frac{\omega_i\,(9-2\omega_i-\omega_j)-\omega_j(6-\omega_j)}{\Omega}.$$

Imposing that $r < \frac{\lambda(3-\Omega)^2}{2}$ (see Lemma 1), we find that $\frac{(3-\Omega)^2}{2} < \theta_{x,i}$, and hence $\frac{r}{\lambda} < \frac{(3-\Omega)^2}{2} < \theta_{x,i}$, which implies that $x_i^*(\omega_i;\omega_j) < x_i^*(0;0)$. For firm j, we find that $\theta_{x,j} < \frac{(3-\Omega)^2}{2}$, and there are two cases: (i) if $\frac{r}{\lambda} < \theta_{x,j} < \frac{(3-\Omega)^2}{2}$ then $x_j^*(\omega_i;\omega_j) < x_j^*(0;0)$; (ii) if $\theta_{x,j} < \frac{r}{\lambda} < \frac{(3-\Omega)^2}{2}$ then $x_j^*(\omega_i;\omega_j) > x_j^*(0;0)$. For a given $\omega_j = \omega_0 > 0$, we can show that $\frac{\partial x_j^*(\omega_i;\omega_0)}{\partial \omega_i} > 0$, and we know that $x_j^*(\omega_0;\omega_0) < x_j^*(0;0)$. Hence, if ω_i is small enough, then (i) is more likely to occur, while if ω_i is sufficiently large then $x_j^*(\omega_i;\omega_j) > x_j^*(0;0)$.

With regards to prices, we find that:

$$p_i^*(\omega_i;\omega_j) - p_i^*(0;0) = \frac{t}{(1-\Omega)\,[t\lambda\,(3-\Omega)^2-2\rho^2]}\,\{t\lambda\,(3-\Omega)\,[2\omega_i(1-\omega_i)+\omega_j(1-\omega_j)+\omega_i\omega_j]$$
$$-\rho^2\,[\omega_i\,(1-\omega_i)\,(3-\omega_i)+\omega_i\omega_j\,(3-\omega_j)-\omega_j\,(1-\omega_j)]\}$$

and

$$sign\{p_i^*(\omega_i;\omega_j) - p_i^*(0;0)\} = sign\{\frac{1}{r} - \frac{\theta_{p,i}(\omega_i,\omega_j)}{\lambda}\},$$

where

$$\theta_{p,i}(\omega_i,\omega_j) \equiv \frac{\omega_i\,(1-\omega_i)\,(3-\omega_i) + \omega_i\omega_j\,(3-\omega_j) - \omega_j\,(1-\omega_j)}{(3-\Omega)\,[2\omega_i(1-\omega_i) + \omega_j(1-\omega_j) + \omega_i\omega_j]}.$$

For firm i, we note that if $\theta_{p,i} < 0$ then $p_i^*(\omega_i;\omega_j) - p_i^*(0;0) > 0$, while if $\theta_{p,i} > 0$ then $\frac{1}{\theta_{p,i}} < \frac{(3-\Omega)^2}{2}$. Since $\frac{r}{\lambda} < \frac{(3-\Omega)^2}{2}$, there are two cases: (i) if $\frac{r}{\lambda} < \frac{1}{\theta_{p,i}} < \frac{(3-\Omega)^2}{2}$ then $p_i^*(\omega_i;\omega_j) > p_i^*(0;0)$; (ii) if $\frac{1}{\theta_{p,i}} < \frac{r}{\lambda} < \frac{(3-\Omega)^2}{2}$ then $p_i^*(\omega_i;\omega_j) < p_i^*(0;0)$. For firm j we find that $p_j^*(\omega_i;\omega_j) > p_j^*(0;0) > 0$ is always satisfied since: (i) if $\frac{1}{\theta_{p,j}} > 0$ then $\frac{(3-\Omega)^2}{2} < \frac{1}{\theta_{p,j}}$, hence $\frac{r}{\lambda} < \frac{(3-\Omega)^2}{2} < \frac{1}{\theta_{p,j}}$; (ii) if $\frac{1}{\theta_{p,j}} < 0$, then $\frac{\theta_{p,j}(\omega_i,\omega_j)}{\lambda} < \frac{1}{r}$, and the result holds. In terms of market shares:

$$s_i^*(\omega_i;\omega_j) - s_i^*(0;0) = \frac{(\omega_j-\omega_i)\left[t\lambda\,(3-\Omega)+\rho^2\right]}{2\left[t\lambda\,(3-\Omega)^2 - 2\rho^2\right]},$$

hence $sign\{s_i^*(\omega_i;\omega_j) - s_i^*(0;0)\} = sign\{\omega_j - \omega_i\}$. If $\omega_i > \omega_j$ then $s_i^*(\omega_i;\omega_j) < s_i^*(0;0)$ and $s_j^*(\omega_i;\omega_j) > s_j^*(0;0)$. The comparison of the special cases follows immediately. ■

APPENDIX D

Here we conduct numerical simulations. To that end, we consider parameter values for which stability and second-order conditions hold, and equilibrium prices, R&D investments, and market shares are positive. To illustrate that total surplus and consumer surplus can increase with asymmetric partial ownership structures we consider some particular cases.

In Figures 1 and 2, we consider the particular case: $\omega_i > \omega_j = 0$. The blue area represents the values for λ and ρ where $TS(\omega_i, 0) > TS(0, 0)$, and the green area represents the values for λ and ρ where $CS(\omega_i, 0) > CS(0, 0)$.

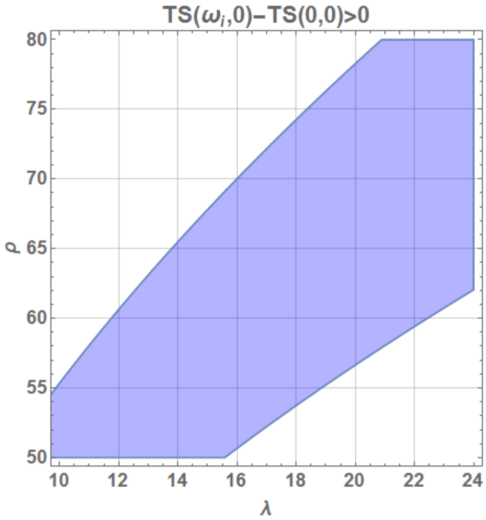

Fig. 1a. $x_0 = 100$, $t = 80$, $\omega_i = 0.1$, $\omega_j = 0$.

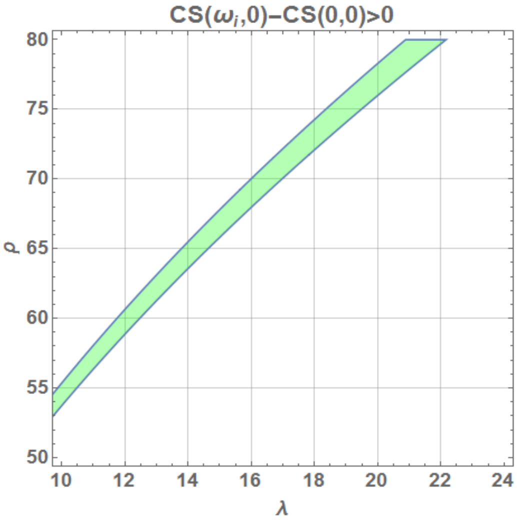

Fig. 1b. $x_0 = 100$, $t = 80$, $\omega_i = 0.1$, $\omega_j = 0$.

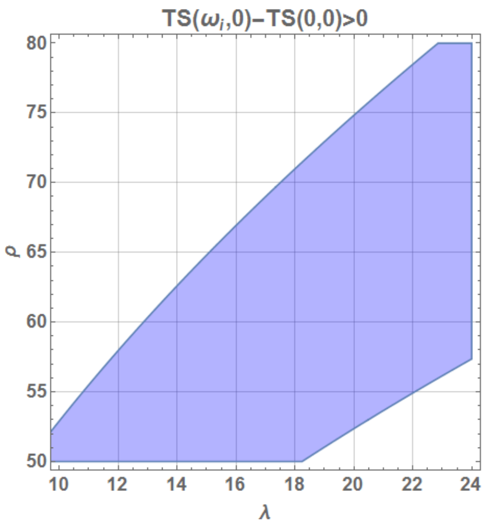

Fig. 2a. $x_0 = 100$, $t = 80$, $\omega_i = 0.15$, $\omega_j = 0$.

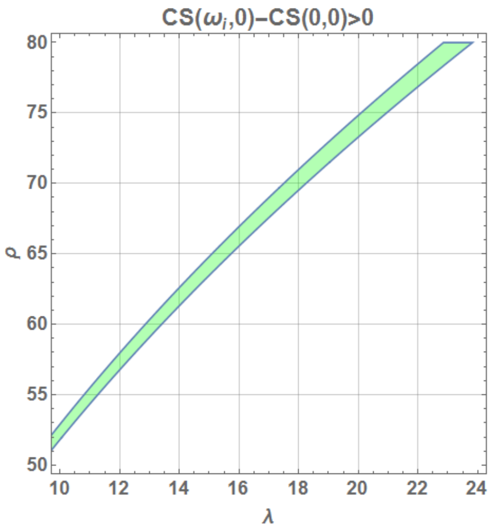

Fig. 2b. $x_0 = 100$, $t = 80$, $\omega_i = 0.15$, $\omega_j = 0$.

In Figures 3 and 4 we consider the general case: $\omega_i > \omega_j > 0$. As above, the blue area represents the values for λ and ρ where $TS(\omega_i, \omega_j) > TS(0,0)$, and the green area represents the values for λ and ρ where $CS(\omega_i, \omega_j) > CS(0,0)$.

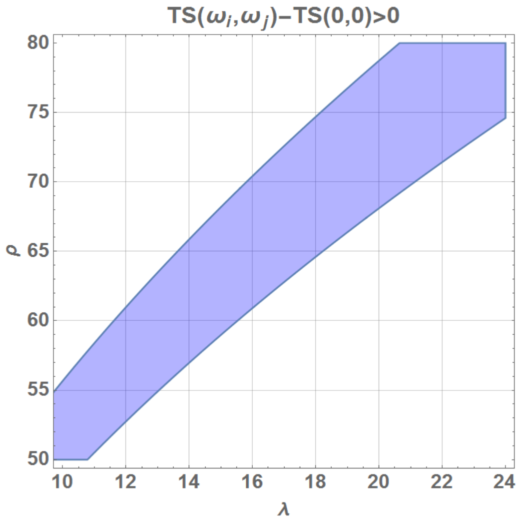

Fig. 3a. $x_0 = 100$, $t = 80$, $\omega_i = 0.1$, $\omega_j = 0.05$.

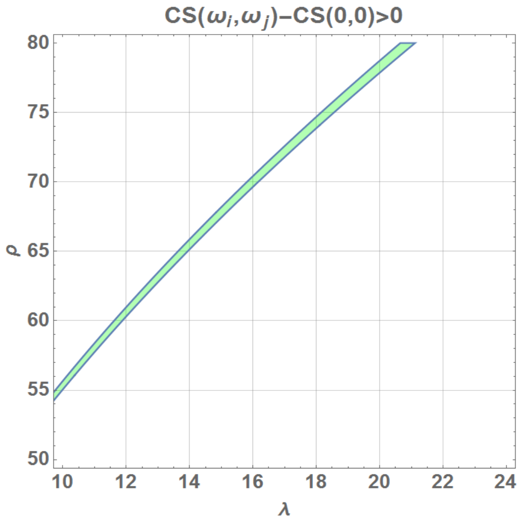

Fig. 3b. $x_0 = 100$, $t = 80$, $\omega_i = 0.1$, $\omega_j = 0.05$.

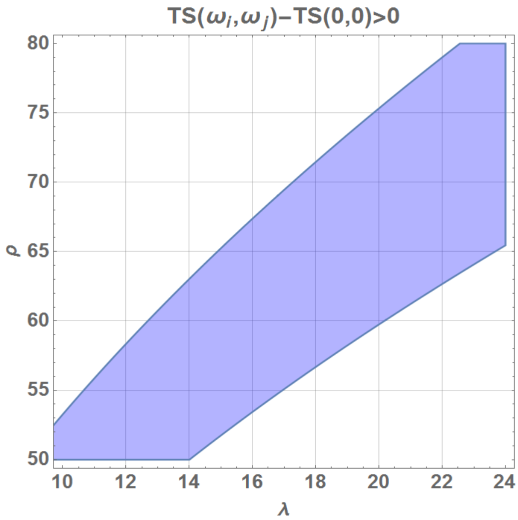

Fig. 4a. $x_0 = 100$, $t = 80$, $\omega_i = 0.15$, $\omega_j = 0.05$.

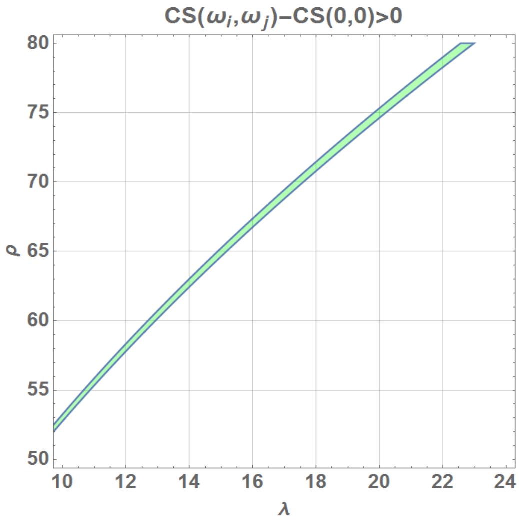

Fig. 4b. $x_0 = 100$, $t = 80$, $\omega_i = 0.15$, $\omega_j = 0.05$.

Note that in Figures 1-4, the ratio $r \equiv \rho^2/t \in [31.25, 80]$.

Finally, next figures depict for given parameter values x_0, t, λ and ρ, the region where $CS(\omega_i, \omega_j) > CS(0, 0)$ for different combinations of ω_i and ω_j.

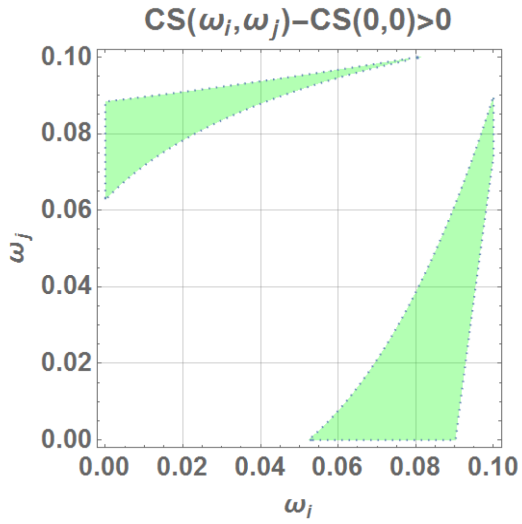

Fig. 5a. $x_0 = 100$, $\lambda = 11$, $t = 84$, $\rho = 60$.

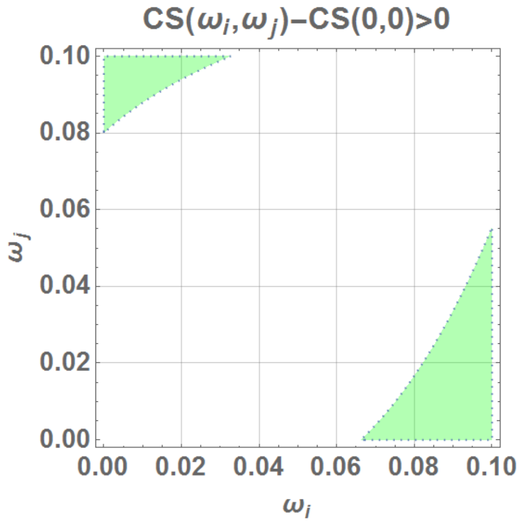

Fig. 5b. $x_0 = 100$, $\lambda = 11$, $t = 86$, $\rho = 60$.

Note that in Figure 5a, $r = 42.85$, whereas in Figure 5b, $r = 41.86$.