The Social Value of Information with an Endogenous Public Signal

Anna Bayona*

September 2017

Abstract

This paper analyses the equilibrium and welfare properties of an economy characterized by uncertainty and payoff externalities using a general model that nests several applications. Agents receive a private signal and an *endogenous public signal,* which is a noisy aggregate of individual actions and causes an information externality. Agents in equilibrium underweight private information for a larger payoff parameter region in relation to when public information is exogenous. In addition, the socially optimal endogenous degree of coordination is lower than the socially optimal exogenous degree of coordination. The welfare effect of increasing the precision of the noise in the public signal has the same sign with endogenous or exogenous public information, but its magnitude differs. The social value of private information may be overturned in relation to when public information is exogenous: from positive to negative if agents in equilibrium coordinate more than is implied by the socially optimal exogenous degree of coordination, and the opposite if they coordinate less.

Keywords: payoff externalities; information externalities; rational expectations equilibrium; private information; welfare analysis.

JEL: D62; D82; G14.

*ESADE Business School. Corresponding e-mail address: *anna.bayona@esade.edu.* I thank the editor, the associate editor and two anonymous referees for providing very useful comments which have greatly improved the paper. I am grateful to my thesis advisor, Xavier Vives, for the guidance and feedback. I would also like to thank Jose Apesteguia, Ariadna Dumitrescu, Emre Ekinci, Juan Imbett, Sebastian Harris, Angel L. Lopez, Carolina Manzano, Margaret Meyer, Manuel Mueller-Frank, Rosemarie Nagel, Morten Olsen, and Alessandro Pavan for constructive comments.

1 Introduction

In today's information age, systems and platforms that collect, aggregate, and process dispersed data about economic agents' strategies in nearly real-time are widespread and broadly accessible at a negligible cost. At the same time, agents' strategies are dependent on the public information that these systems aggregate.[1] In other words, public information is endogenous and agents increasingly use strategies that are contingent on noisy statistics that aggregate the actions of other agents.

In this context it is relevant to ask whether providing more precise information is socially valuable. This concerns policymakers in two respects. First, under what conditions does greater transparency improves social welfare? Second, is it in fact desirable to facilitate tools that improve the private observation of local market conditions? In this paper, I study the introduction of an endogenous public signal into economies with uncertainty and payoff-externalities. This brings new insights to the existing literature and enables a clear distinction to be made between environments with endogenous and exogenous public information.

I present two motivating arguments which compare these two types of public information. The first is related to whether public information either monitors fundamentals directly (exogenous), or whether it tracks market activity (endogenous). With the prevalence of big data, there are nowadays many indicators of economic activity that provide real-time forecasts to predict the present (see Choi and Varian 2012), and which therefore constitute an endogenous source of public information. In these environments, agents use schedules that are contingent on the noisy contemporaneous public forecast about the aggregate action. For example, firms that make capacity choices are often uncertain about the shocks that affect their demand. The increasing prevalence of online platforms facilitates the aggregation of information about the strategies of rivals. In turn, firms post schedules in which the capacity they offer is contingent on noisy public information about the average capacity offered in the market and on their own private information. The second motivation relates to whether the interaction among agents occurs in segmented markets, where public information is exogenous (e.g., in over-the-counter markets), or in centralized markets that facilitate information aggregation, where public information is endogenous (e.g., in exchanges).[2]

Three research questions emerge. First, how do agents make optimal use of the different types of information sources in equilibrium? Second, what are the welfare properties of the equilibrium strategy? Third, what are the effects on welfare of varying the precision of public and private information? The main finding is that endogenous public information can overturn the social value of private information in relation to when public information

[1] See, for example, McAfee et al. (2012) and Einav and Levin (2014).

[2] See, for example, Avdjiev, McGuire and Tarashev (2012).

is exogenous. The paper finds that the interaction of information and payoff externalities characterize the direction and size of the effects that endogenous public information has on the social value of private information.

I address these questions by using a static model of the linear-quadratic Gaussian family based on the payoff structure of Angeletos and Pavan (2007). The economy is populated by a large number of agents. Each agent's utility function depends on fundamentals and exhibits payoff externalities: strategic complementarity or substitutability. Agents have access to a private signal and to an endogenous public signal, which is a noisy aggregate of individual strategies. The equilibrium concept used is the (linear) rational expectations equilibrium. The welfare benchmark is the ex ante utility of a team of agents subject to the constraint that private information cannot be transferred from one agent to another, or to a center. The model is general and can incorporate a number of applications, such as competition in a homogeneous product market; competition à la Bertrand or à la Cournot; and the beauty contest.

The unique linear equilibrium strategy exhibits the following property. When public information aggregates the economy's dispersed information, the precision of the endogenous public signal depends quadratically on the agents' response to private information. Agents in equilibrium do not take into account that their response to private information affects the informativeness of the endogenous public signal. Hence, endogenous public information always generates an information externality. In addition, there are two further possible types of externalities: first, with full information agents may over- or under- respond to the fundamental; and second, with incomplete information, agents may perceive a higher or lower degree of coordination in relation to that perceived by the welfare planner. The combination of these characterizes whether agents over-, under-, or equally weight private information in relation to the efficient strategy. For example, in competition à la Bertrand with product differentiation, firms may underweight private information in relation to the efficient level, thereby overturning the conclusions with exogenous public information. As a consequence, the socially optimal endogenous degree of coordination is lower than the socially optimal exogenous degree of coordination. In addition, the characteristics of the payoff structure alone are not sufficient to ensure efficiency. The properties of the information structure need to be taken into account.

The wedge between equilibrium and efficient strategies generates a welfare loss consisting of various components, which jointly influence the social value of information. The welfare loss consists of a first order welfare effect, which is related to the sign and magnitude of the full-information inefficiency. It also consists of two second order effects due to incomplete information generated by the noise in private and public signals, dispersion and volatility, respectively. I find the necessary and sufficient conditions which determine the social value of information with an endogenous public signal.

The social value of private information with an endogenous public signal may change sign and magnitude in relation to when public information is exogenous. This is because its social value is related to whether public information is beneficial or detrimental (since changing the precision of the private signal increases the overall precision of the endogenous public signal, which has an additional welfare effect). The social value of private information with an endogenous public signal depends crucially on how the three potential inefficiencies of equilibrium strategy combine.

In contrast, the introduction of an endogenous public signal affects only the magnitude of the social value of public information. The reason for this is that higher precision of the noise in the endogenous public signal always increases the overall precision of the endogenous public signal. This is because the positive accuracy effect (the signal is more informative) dominates the negative crowding-out effect (which reduces the weight that agents place on the private signal thus making the endogenous public signal less informative).

Combining the social value of private and (endogenous) public information, I characterize all the possible sign combinations of the various welfare effects in relation to the primitives of the model. First, I consider the benchmark case where there are no inefficiencies with either complete or incomplete information (e.g., firms that compete in a homogeneous product market with total surplus as a welfare benchmark). In this case, with both exogenous and endogenous public information, more precise public or private information is beneficial for welfare. However, the magnitude of the change in the social value of private information, that results from an increase in the precision of the private signal, is larger with endogenous than with exogenous public information.

To understand the intuition of the main result, consider the application of monopolistic competition à la Bertrand, in which the full information inefficiency is small and firms slightly under-respond to the fundamental. With incomplete and exogenous information, firms give too much weight to private information since they coordinate less than is efficient. Hence, more precise public information is always welfare improving since it helps agents reduce the excess equilibrium weight on private information. This is reflected in equilibrium welfare losses since more precise public information decreases dispersion and the first order effect more than the potential increase in volatility. In contrast, more precise private information may in fact reduce social welfare when public information is either exogenous or endogenous. Due to the endogenous public signal, the welfare effect of increasing the precision of the private signal may be overturned from negative to positive. This is because a higher precision of the private signal increases the overall precision of the endogenous public signal. Thus agents give greater weight to the public signal, which further reduces dispersion. Since dispersion has a large relative weight in welfare loss, the overall effect of the endogenous public signal on the social value of private information is positive.

In economies where agents with full information over-respond to the fundamental, then increasing the precision of either public or private information increases equilibrium welfare loss through the first order effect. Hence, the benefits of more precise either public or private information become smaller in relation to economies that are efficient with full information. At the limit, when the full information inefficiency is very large, more precise private and public information always diminish welfare.

Considering an exogenous information structure, Ui and Yoshizawa (2015) categorize the different welfare effects of information into eight types of games. I study how endogenous public information changes their categorization. There are three differences in the social value of information due to the endogenous public signal. First, the payoff parameters alone do not always characterize the social value of information since the characteristics of the information structure matter. Second, some of the cutoffs between types of game change (those that depend on the parameters related to the social value of private information). Third, there is a change in cutoffs within the types of game where the sign of the social value of information depends on the ratio of public to private precisions. To sum up, the social value of private information may be overturned in relation to when public information is exogenous: from positive to negative if agents in equilibrium coordinate more than is implied by the socially optimal exogenous degree of coordination, and the opposite if they coordinate less.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 describes the model. Section 4 studies the equilibrium and efficient strategies. Section 5 presents the results of the social value of information. Section 6 illustrates the main results of the paper in two applications. Section 7 concludes. Proofs are derived in the Appendix.

2 Related Literature

This paper is the first to show how payoff inefficiencies combine with the information externality through studying the social value of information in a general type of quadratic payoff structures with incomplete information. There are two relevant strands of literature. The first concerns the efficiency in the use and the social value of exogenous information sources. The second deals with the role of endogenous information structures in a variety of settings.

The social value of information with exogenous information in linear-quadratic-normal games has been studied previously by Morris and Shin (2002), in the beauty contest; by

Hellwig (2005), in a business cycle framework; by Angeletos and Pavan (2007), henceforth AP, in a general model which encompasses many of the previous applications; and by Myatt and Wallace (2015, 2017), in models of differentiated product oligopolies with rich industry features.[3] Ui and Yoshizawa (2015), henceforth UY, categorize all of these games mentioned previously into types according to the properties regarding the social value of information. In relation to this literature, I first show how payoff and information externalities combine in the use of information. Second, I show how introducing an endogenous public signal may modify the sign of the social value of private information. In consequence, this changes both the threshold value of payoff parameters that define the type of game, and also the ratio of public to private precisions that determines the sign of the social value of information.

The information environment is reminiscent of the literature of social learning such as in Banerjee (1992) and Bikhchandani et al. (1992), which study the welfare effects of pure information externalities where agents respond insufficiently to private signals. Papers in the rational expectations tradition are also related, which typically consider static models where the public signal has both information and allocation roles and agent's strategies are schedules (e.g., Grossman and Stiglitz 1980; Diamond and Verrechia 1981). In contrast, I concentrate on environments where the endogenous public signal has an information role but not an allocation role since it does not directly affect an agent's payoff function. The framework that I use in this paper proposes the simplest model that combines elements of rational expectations, herding, and games of strategic complementarity and substitutability.

The results presented here are related to several papers that focus on the implications for policy and that also consider an endogenous public signal. Morris and Shin (2005) were the first to show that public signals are less informative when central bankers disclose their forecasts than when they do not disclose anything. Angeletos and Pavan (2009) also consider that public information is endogenous and focus on the optimal design of contingent taxation. Amador and Weill (2010) find that more precise public information can be detrimental in a model with no payoff externalities, which contrasts with the results of this paper. Their model contains an additional source of strategic complementarity, generated by the way agents learn from prices, which is responsible for the negative welfare result. Bond and Goldstein (2015) analyze the implications of governments relying on prices to guide their decisions.

This paper also complements the results of research which study costly endogenous information acquisition. Hauk and Hurkens (2001) compare the welfare properties of costly private information acquisition (two-stage game) and secret information acquisition (one-stage game) in Cournot markets. Colombo et al. (2014) study the effects of costly endogenous private information in a general model. They find that the crowding-out effects of public information

[3] Clark and Polborn (2006) also study this issue in a binary choice context.

on private information may overturn the magnitude and sign of the social value of public information in relation to when private information is exogenous. In contrast, this paper focuses on the effects of a costless endogenous public signal on the use of information and on the social value of private information.

3 The model

I present a static linear-quadratic-Gaussian model with an information structure that includes both a private signal and an endogenous public signal.

Preferences The economy is composed of a continuum of agents distributed uniformly over the unit interval and indexed by i. Simultaneously, each agent chooses a strategy, k_i, to maximize the utility function $U(k_i, K, \sigma_k, \theta)$, where θ represents the fundamentals that exogenously affect an agent's utility function; K is the average of agents' actions given by $K = \int k_i di$; and σ_k^2 is the variance of the agents' actions across the population defined as $\sigma_k^2 = \int (k_i - K)^2 di$. I assume that U is a quadratic polynomial in k_i that is symmetric across agents. The dispersion in actions has only second-order effects (i.e., $U_{k\sigma} = U_{K\sigma} = U_{\theta\sigma} = 0$), and $U_\sigma(k_i, K, 0, \theta) = 0$ for all (k, K, θ). Furthermore, the utility function is concave with respect to k_i. Without loss of generality, let $\alpha = -\frac{U_{kK}}{U_{kk}}$ be the degree of strategic complementarity.[4] Actions are strategic complements whenever $\alpha > 0$, strategic substitutes whenever $\alpha < 0$, and strategically independent whenever $\alpha = 0$. To ensure uniqueness, assume that $\alpha < 1$. Assume that $U_{k\theta} \neq 0$, and note that the payoff function described above admits payoff externalities with respect to the mean action whenever $U_K \neq 0$ and with respect to the dispersion of actions whenever $U_\sigma \neq 0$.

Examples of preferences that can be modeled with this framework from the literature are as follows. First consider an economy that is composed of a continuum of households, each consisting of producers and consumers, who make production choices with quadratic production costs in a homogeneous product market. Suppose that agents are uncertain about common fundamentals, θ, which represent the intercept of the marginal cost. Vives (1988) and AP show that households' utility function can be written as $U(k, K, \sigma_k, \theta) = (\delta - \beta K)k_i + \frac{\beta K^2}{2} - \theta k_i - \frac{\lambda k_i^2}{2}$, such that $\beta + \lambda > 0$ and $2\beta + \lambda > 0$. Hence $\alpha = \frac{-\beta}{\lambda}$.

A second application is the Keynesian beauty contest formalized by Morris and Shin (2002), where agents are not only concerned about predicting fundamentals but also about outguessing the likely actions of others. This has been used as a metaphor on how financial

[4] AP refer to α as the equilibrium degree of coordination.

markets work. An agent's utility function can be expressed as $U(k, K, \sigma_k, \theta) = -(1-r)(k_i - \theta)^2 - r(k_i - K)^2 + r\sigma_k^2$, where $\alpha = r \in (0, 1)$.

Two other applications are developed in Section 6, which focus on monopolistic competition à la Bertrand and à la Cournot with product differentiation.

Uncertainty and Information The common demand shock, θ, follows a normal distribution with mean $\bar{\theta}$ and variance σ_θ^2, i.e., $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$. To form expectations about the common shock, each agent receives two types of information. First, agents have access to a noisy private signal about the common shock, $x_i = \theta + \xi_i$, with the noise distributed as $\xi_i \sim N(0, \sigma_\xi^2)$. Second, agents receive a public signal about the aggregate action, which emerges within the economy, and therefore is *endogenous* (see motivation in the introduction). The endogenous public signal is given by

$$w = \int k_i di + v, \tag{1}$$

where the noise in the endogenous public signal is normally distributed with $v \sim N(0, {\sigma_v}^2)$, which reflects the fact that the aggregation process is imperfect.[5] Fundamentals and error terms of private and public signals are mutually independent and identically distributed across agents. For ease of interpretation, I shall often work with the precision of a random variable, which is the inverse of its variance. Throughout the text, τ_ξ is the precision of the noise in the private signal, τ_v the precision of the noise in the endogenous public signal, and τ_θ the precision of fundamentals.

This modeling approach has the advantage of introducing an endogenous public signal with minimal modifications to the standard benchmark of exogenous public information, in which the public signal is typically defined as $y = \theta + \varepsilon$ with $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. Therefore, comparisons between the two environments are straightforward.

Equilibrium Definition Agent i's strategy $k(x_i, .)$ is a mapping from the signal space to the action space of each agent. A rational expectations equilibrium (REE) satisfies:

1. Each agent i chooses a strategy $k(x_i, w)$ that maximizes expected utility $E[u|x_i, w]$ conditional on the information set $\{x_i, w\}$ and on knowing $w(\theta, v)$, as well as the underlying distributions of random variables.

2. The endogenous public signal is formed so that (1) holds.

[5]This is not the only way in which the public signal can be made endogenous. Refer to the Section 2 for a discussion on the alternative formulations used in the literature.

The first condition requires that each agent maximizes expected utility given his information set, which involves extracting from w all its informational content. The second condition enforces the required consistency of rational expectations. This implies that the endogenous public signal both aggregates agents' dispersed actions and at the same time depends on the information which these provide. In other words, the conjectured strategy must be self-fulfilling (fixed point of conjectured strategies to actual strategies). Unless the public signal is infinitely precise, the REE will be partially revealing.[6] Given the linear-quadratic-Gaussian structure of the model, I restrict equilibria in the class of linear strategies, as is usual in the literature.

Discussion of the Equilibrium Concept The REE concept may be problematic; its limitations for Bayesian games have been discussed extensively in the literature. Specifically, in the set-up of this paper, strategies and the endogenous public statistic are determined simultaneously. In essence, this is the same problem that has been studied in the financial economics literature where agents use the informational content of the price (the endogenous public statistic) but they do not consider the influence of their strategies on market clearing (in the model considered here, w, is not part of an agent's utility function).[7] This equilibrium concept is questionable when the number of agents is finite. However, a noisy REE with a continuum of agents, as in this paper, can be shown to overcome many of its problems. For a further discussion of this issue refer to Hellwig (1980), Kovalenkov and Vives (2014), and Vives (2014). An alternative approach would be to consider a dynamic model. The main advantage of the static model over a dynamic one is its greater simplicity, which allows the main insights to be demonstrated more clearly. A similar approach has been used by Angeletos and Werning (2006) in the context of global games where individuals observe one another's actions.

Vives (2014) discusses the equivalence between an implementable REE and a Bayesian Nash equilibrium of a competitive game where each agent submits a schedule and has a demand or supply function interpretation (see Klemperer and Meyer 1989). In the context of this paper, the REE is implementable by considering a game where each agent decides a schedule by positing a reaction function, which relates an agent's strategy to the realization of the endogenous public signal. The timing of such game can be specified as follows. First, fundamentals and errors terms are drawn. Second, agents (who do not observe the fundamentals) receive a private signal and conjecture a schedule that satisfies the maximization condition (1). Third, the public signal is formed and satisfies the consistency condition (2),

[6] If $\tau_v \to \infty$ then the REE would be fully revealing.

[7] Refer to Section 2 for further details of this literature.

and its precision is determined. Finally, payoffs are collected. For a further discussion on the use of REE in market games refer to Shorish (2010).

4 Equilibrium and Efficient strategies

In this section, I present the equilibrium and efficient strategies, describe their properties, and compare them to the benchmark with exogenous public information.

4.1 Equilibrium strategy

To derive the equilibrium with an endogenous public signal, I use the REE definition from the previous section. Focusing on linear arbitrary strategies of the form $k(x_i, z) = b' + ax_i + c'E[\theta \mid z]$, where a is the response to private information. Given the structure of the information environment, I show that the informational content of the endogenous public signal is equivalent to that of $z = a\theta + v$. Hence, a strategy can be more concisely written as $k(x_i, z) = b + ax_i + cE[\theta|z]$. Owing to the properties of normal distribution: $E[\theta|z] = \frac{\tau_v az + \tau_\theta \bar{\theta}}{\tau_\theta + a^2\tau_v}$ and $(var[\theta|z])^{-1} = \tau(a) = \tau_\theta + a^2\tau_v$, the latter of which is the *overall precision of the endogenous public signal.*

With full information, AP show that there is a unique equilibrium which satisfies $k_i = K = \kappa(\theta) = \kappa_0 + \kappa_1\theta$ for all i, where $\kappa_0 = \frac{-U_k(0,0,0,0)}{U_{kk}+U_{kK}}$ and $\kappa_1 = \frac{-U_{k\theta}}{U_{kk}+U_{kK}}$. Using these considerations, Proposition 1 spells out the equilibrium strategy when public information is endogenous. In what follows, the precisions of any exogenous noise are always non-zero and finite.

Proposition 1

There exists a unique linear REE strategy which is given by

$$k^m(x_i, z) = \kappa_0 + \kappa_1(\gamma^m x_i + (1 - \gamma^m)E[\theta|z]), \tag{2}$$

where the equilibrium weight to private information, $\gamma^m \in (0, 1)$, *is the unique real root of the implicit equation* $\gamma = \frac{(1-\alpha)\tau_\xi}{(1-\alpha)\tau_\xi + \tau_\theta + \kappa_1^2\gamma^2\tau_v}$.

Let us define the precision of the endogenous public signal at the equilibrium strategy as: $\tau^m = \tau(\kappa_1\gamma^m) = \tau_\theta + (\kappa_1\gamma^m)^2\tau_v$. Endogenous public information modifies the precision of the public signal in relation to the case of exogenous public information since, as agents respond more to private information, the more informative the public signal becomes. This feedback effect between an individual strategy and the precision of the endogenous public signal drives the main results of the paper. The precision of the endogenous public signal affects the size of the Bayesian weight, which modifies the equilibrium weights an agent gives to public and private information. This effect is not present when public information is exogenous since the equilibrium strategy derived by AP is $k_{exo}(x_i, z) = \kappa_0 + \kappa_1(\gamma_{exo}x_i + (1 - \gamma_{exo})y)$, where $\gamma_{exo} = \frac{(1-\alpha)\tau_\xi}{(1-\alpha)\tau_\xi+\tau_\theta+\tau_\varepsilon}$, and the overall precision of the exogenous public signal is $\tau_\theta + \tau_\varepsilon$.[8]

The following corollary states comparative statics of the equilibrium weight to private information with respect to information and payoff relevant parameters.

Corollary 1

The comparative statics of the equilibrium weight given to the private signal, γ^m, are

$$\frac{\partial \gamma^m}{\partial \tau_\xi} > 0, \frac{\partial \gamma^m}{\partial \tau_\theta} < 0, \frac{\partial \gamma^m}{\partial \tau_v} < 0, \text{ and } \frac{\partial \gamma^m}{\partial \alpha} < 0. \tag{3}$$

When the precision of the noise in the private signal tends to zero, agents do not give any weight to this signal. As the precision of the private signal increases, agents increase the weight given to the private signal. As τ_θ or τ_v increase, the public signal becomes more informative and agents shift their weight from the private to the public signal. At the limit, when the noise in the public signal is infinitely precise, agents give all the weight to the public signal and the equilibrium collapses. As in the case of exogenous public information, if actions were strategically independent ($\alpha = 0$), the weights given to public and private information would correspond to Bayesian weights according to the relative precisions of the signals. When actions are strategic complements ($\alpha > 0$) agents place greater weight on the public signal since this helps to better predict the aggregate action. The converse happens when actions are strategic substitutes ($\alpha < 0$). These comparative statics have important implications on how τ^m changes with the information parameters as shown below.

Corollary 2

The effects of changing $\tau_\xi, \tau_\theta, \tau_v$ on the overall precision of the public signal at the equilibrium strategy, τ^m, are

$$\frac{d\tau^m}{d\tau_\xi} > 0, \frac{d\tau^m}{d\tau_\theta} > 0, \text{ and } \frac{d\tau^m}{d\tau_v} > 0. \tag{4}$$

[8] The paper closely follows AP's notation, except from the weight to private information. In this paper, the weight to private information with an *endogenous public signal* is γ^m, while the weight to private information with an *exogenous public signal* is γ_{exo}. AP use $1 - \gamma$ to denote the weight to private information with an exogenous public signal. The reason for this alternative notation is that the mathematical expressions become significantly more compact since most of the results throughout the paper depend primarily upon the weight given to private information.

Increasing τ_ξ makes agents place more weight on the private signal, γ^m, which increases the overall informativeness of the endogenous public signal, τ^m.

However, increasing τ_v has two effects on τ^m, which are of opposite sign. First, for a fixed weight on the private signal, increasing the precision of the noise in the public signal raises the overall precision of the endogenous public signal since information is more accurate (*accuracy effect*). Second, it reduces the weight that agents place on the private signal, which decreases the informativeness of the endogenous public signal (*crowding-out effect*). I find that the accuracy effect always dominates the crowding-out effect, and hence and $\frac{d\tau^m}{d\tau_v} > 0$.[9]

4.2 Efficient strategy

Welfare analysis requires a definition of the welfare benchmark appropriate for this setting. I follow the approach of Radner (1962), Vives (1988), and AP who consider that there is a welfare planner that maximizes the ex ante utility subject to the constraint that information cannot be transferred from one agent to another, and that the welfare planner does not have access to agents' private information. Let us define social welfare as $W(K, \sigma_k, \theta) = \int U(k_i, K, \sigma_k, \theta)di$ such that $W_{KK} < 0$ and $W_{\sigma\sigma} < 0$.

AP show that with full information there is a unique efficient strategy which satisfies $k_i^* = K^* = \kappa^*(\theta) = \kappa_0^* + \kappa_1^*\theta$, where $\kappa_0^* = \frac{-W_K(0,0,0)}{W_{KK}}$ and $\kappa_1^* = \frac{-W_{K\theta}}{W_{KK}}$. With incomplete information, the ex ante utility for a candidate efficient strategy is then

$$E[u] = E[W(\kappa^*(\theta), 0, \theta)] + \frac{W_{KK}}{2} E\left[(K - \kappa^*(\theta))^2\right] + \frac{W_{\sigma\sigma}}{2} E\left[(K - k_i)^2\right], \tag{5}$$

which is the sum of the following terms: the expected aggregate utility at the full information efficient strategy, which only depends only on τ_θ, and is the first best allocation; non-fundamental volatility; and dispersion. Expected welfare loss is $L^* = E[W(\kappa^*(\theta), 0, \theta)] - E[u]$, and the trade-off between volatility and dispersion is $\frac{W_{KK}}{W_{\sigma\sigma}} = 1 - \alpha^*$. AP define $\alpha^* < 1$ as the socially optimal degree of coordination. Here after I shall refer to α^* as the socially optimal *exogenous* degree of coordination (if $\alpha^* > 0$ then it is efficient that agents perceive strategies as strategic complements; if $\alpha^* < 0$ as strategic substitutes; and if $\alpha^* = 0$ as independent). [10]When public information is endogenous, the efficient strategy $k^*(x_i, z)$ can be found by maximizing the ex ante utility and is given in the next proposition.

[9] This discussion also applies to the logic of the comparative statics with respect to τ_θ.

[10] I have added the word exogenous to "socially optimal *exogenous* degree of strategic coordination" since α^* is no longer optimal with an endogenous public signal. Notice that α^* only depends on the parameters of the utility function. For further details refer to Proposition 4.

Proposition 2

The efficient linear strategy under incomplete information is

$$k^*(x_i, z) = \kappa_0^* + \kappa_1^*(\gamma^* x_i + (1-\gamma^*)E[\theta \mid z]), \tag{6}$$

where the efficient weight to private information, $\gamma^* \in (0,1)$*, is the unique real root of the implicit equation* $\gamma = \frac{(1-\alpha^*)\tau_\xi}{(1-\alpha^*)\tau_\xi + \tau_\theta + \kappa_1^{*2}\tau_v\gamma^2 - (1-\alpha^*)(1-\gamma)^2 \frac{\tau_v \tau_\xi \kappa_1^{*2}}{(\tau_\theta + \kappa_1^{*2}\tau_v\gamma^2)}}$.

The welfare planner balances the effects of an increase in the weight to private information on the components of expected welfare loss since: (1) it increases dispersion; (2) it decreases non-fundamental volatility and, (3) due to endogenous public information, it increases the overall precision of the endogenous public signal (which affects the level of strategic uncertainty) and further reduces volatility. Taking these three effects into account, the welfare planner finds the unique γ^* which minimizes expected welfare loss.

The comparison of the efficient and equilibrium strategies shows the three types of inefficiencies which may be present at the equilibrium strategy. First, there may be a full information inefficiency if $\kappa^*(\theta) \neq \kappa(\theta)$. Second, keeping the precision of the public signal fixed, differences between α and α^* mean that agents do not have the same individual incentives to coordinate their actions in relation to the efficient level. These first two inefficiencies were first analyzed by AP and are reflected in the efficient strategy with exogenous public information given by $k^*_{exo}(x_i, z) = \kappa_0^* + \kappa_1^*(\gamma^*_{exo} x_i + (1-\gamma^*_{exo})y)$, where $\gamma^*_{exo} = \frac{(1-\alpha^*)\tau_\xi}{(1-\alpha^*)\tau_\xi + \tau_\theta + \tau_\varepsilon}$.

The third inefficiency, which is the core of this paper, is due to the endogeneity of the public signal. It is reflected in the denominator of the efficient weight to private information, which includes an additional term in relation to when public information is exogenous. In contrast to the welfare planner, agents in equilibrium do not take into account that a higher weight on the private signal increases the precision of the endogenous public signal, thus generating an information externality. This information externality is always present and depends on both payoff and information parameters. This is a major difference with respect to the first and second sources of externalities, which only depend on the payoff structure, and not on the superimposed information structure. Notice that the information externality vanishes if the noise in the endogenous public signal or the private signal are infinitely volatile (i.e., $\tau_v \to 0$ or $\tau_\xi \to 0$). The next corollary derives the comparative statics of the γ^* with respect to information and payoff parameters.

Corollary 3

The comparative statics of the efficient weight given to the private signal, γ^*, are

$$\frac{\partial \gamma^*}{\partial \tau_\xi} > 0, \frac{\partial \gamma^*}{\partial \tau_\theta} < 0, \text{ and } \frac{\partial \gamma^*}{\partial \alpha^*} < 0. \tag{7}$$

The comparative statics of γ^* with respect to τ_v are ambiguous.

The efficient weight on the private signal increases with τ_ξ, whereas it decreases with τ_θ and with α^* (for similar reasons to the ones given in Corollary 2). However, the comparative statics of γ^* with respect to the precision of the noise in the endogenous public signal are ambiguous and different from the case of exogenous public information (where a higher precision of the noise in the exogenous public signal always decreases the efficient weight on x_i). In addition to the effect with exogenous public information, the planner takes into account that, due to the information externality, increasing τ_v has a crowding-out effect which requires a higher weight on the private signal. Hence the ambiguous comparative statics of γ^* with respect to τ_v follow.

Proposition 3 shows how the information externality combines with payoff externalities due to incomplete information in establishing if agents over- or under- weight private information in relation to the efficient weight strategy.

Proposition 3

Define $\hat{\alpha} = \alpha^* - \frac{(1-\alpha^*)(1-\alpha)(\kappa_1^2 \tau_v \tau_\xi)}{((1-\alpha)\tau_\xi + \tau^m)^2}$. The sign of the difference between the equilibrium and efficient weights given to private information is

$$sign\left(\gamma^m - \gamma^*\right) = sign\left(\hat{\alpha} - \alpha\right). \tag{8}$$

Endogenous public information breaks the alignment between $\alpha^* - \alpha$ and $\gamma^m - \gamma^*$ that exists when public information is exogenous since AP show that: $sign\left(\gamma_{exo} - \gamma_{exo}^*\right) = sign\left(\alpha^* - \alpha\right)$. The comparison of the efficient weights to private information with endogenous and exogenous public signals is as follows. Denote the precision of the endogenous public signal at the efficient strategy found in Proposition 2 as $\tau^* = \tau(\kappa_1^* \gamma^*) = \tau_\theta + \tau_v(\kappa_1^* \gamma^*)^2$. Take an exogenous public signal with noise precision $\tau_\varepsilon = \tau^* - \tau_\theta$. Then the efficient weight on private information under such exogenous public signal is always smaller than γ^*.

Economies with $\alpha = \alpha^*$ represent an important benchmark. If public information was exogenous then the equilibrium weight to private information would always be efficient. However, with endogenous public information, agents give an inefficiently low weight to private information for all the possible parameter combinations of the payoff function and endogenous information structures. This is because agents do not take into account how their individual actions affect the precision of the endogenous public signal. An example of an economy with such structure is given when firms compete in a homogeneous product market with total surplus as welfare benchmark described in Section 3. In this application: $\alpha^* = \alpha$ and $\kappa = \kappa^*$.[11]

Due to the additional coordination role of the endogenous public signal, if $\alpha^* > \alpha$ then the information externality may overturn $sign\,(\gamma^m - \gamma^*)$ in relation to $sign\,(\gamma_{exo} - \gamma^*_{exo})$. In fact, when the information externality is sufficiently large, agents may equally or underweight private information in relation to the efficient level. This contrasts with the result with exogenous public information since agents in equilibrium always give too much weight to private information. Consequently, with endogenous public information, agents in equilibrium may underweight private information for a larger payoff parameter region in relation to when public information is exogenous. In Section 6, I show how this may occur in competition à la Bertrand with product differentiation.

In contrast, if $\alpha > \alpha^*$ then agents always give too little weight to private information in relation to the efficient level. However, the difference in magnitude between the efficient and equilibrium weights to private information is larger with endogenous than with exogenous public information.[12] For example, this occurs in the beauty contest described in Section 3 since its payoff structure satisfies $\alpha = r > 0 = \alpha^*$.

Proposition 4 states the necessary and sufficient conditions for an economy and information structure to be jointly efficient.[13]

Proposition 4

An economy characterized by $U(k, K, \sigma_k, \theta)$ and an information structure $(\tau_\xi, \tau_\theta, \tau_v)$, such that $w = \int k_i di + v$, are jointly efficient if and only if

$$\kappa(\theta) = \kappa^*(\theta),\ \forall\, \theta,\ \textit{and}$$
$$\alpha = \hat{\alpha} < \alpha^*.$$

Proposition 4 emphasizes that, with an endogenous public signal, the efficiency condition

[11]This result has also been found in Bru and Vives (2002) focus on a pure prediction ($\alpha^* = \alpha = 0$) with endogenous public information. Models that consider a payoff structure with $\alpha = \alpha^*$ but in which price has an additional allocation role, such as in Vives (2017), and where agents use price contingent strategies, lead to different results as those presented in Proposition 3. For example, agents in equilibrium can give too much weight to private information whenever actions are strategic substitutes since the market price has dual information and allocation roles.

[12]An implication is that the equilibrium precision of the endogenous public signal, τ^m, may be too high or too low in relation to the efficient level, τ^*.

[13]I thank a referee for suggesting that this result should be discussed.

requires both the payoff and information structures to be taken into account. This contrasts with the efficiency result with exogenous public information which only depends on the characteristics of the payoff structure: AP find that an economy is efficient if and only if $\kappa(\theta) = \kappa^*(\theta)$ for all θ, and $\alpha = \alpha^*$. With endogenous public information the former payoff conditions are not sufficient for efficiency: the characteristics of the information structure are essential (recall the definition of $\hat{\alpha}$ in Proposition 3). Furthermore, the socially optimal endogenous degree of coordination, $\hat{\alpha}$, is lower than the socially optimal exogenous degree of coordination, α^*. This is because the endogenous public statistic, w, provides further information about the strategies of other players, thus fostering an additional equilibrium degree of coordination, which is corrected by the welfare planner.

5 Social Value of Public and Private Information

In this section, I analyze whether more precise public and private signals have positive or negative welfare effects, and focus on the differences in environments with endogenous and exogenous public information.

Ex ante utility can be expressed as $E[u] = E\left[W(\kappa(\theta), 0, \theta)\right] - WL$, where $WL = -\frac{W_{\sigma\sigma}}{2} E\left[(K-k)^2\right] - \frac{W_{KK}}{2} E\left[(K-\kappa(\theta))^2\right] - E[W_K(\kappa(\theta), 0, \theta)(K-\kappa(\theta))]$. The first two terms are second-order effects: the first term is dispersion and the second is non-fundamental volatility. The third term is a first order effect, which is related to the full information inefficiency, and is the covariance between marginal social welfare, $W_K(\kappa(\theta), 0, \theta)$, and the aggregate error due to incomplete information, $K - \kappa(\theta)$. AP show that substituting for the equilibrium strategy in WL gives

$$WL = \frac{\mid W_{\sigma\sigma} \mid \kappa_1^2}{2} \left(\frac{(\gamma^m)^2}{\tau_\xi} + \frac{(1-\alpha^*)(1-\gamma^m)^2}{\tau^m} + \frac{2\phi(1-\alpha^*)(1-\gamma^m)}{\tau^m} \right), \tag{9}$$

where $\phi = \frac{\kappa_1^* - \kappa_1}{\kappa_1}$ is the full information inefficiency ratio defined by AP.[14] This last term of (9) will be henceforth abbreviated as *first order effect in* WL.

UY use the same measure of welfare but represent it differently, expressing it as a linear combination of idiosyncratic variance and common variance. The idiosyncratic variance corresponds to the dispersion term, while the common variance is different and is defined as the covariance of actions. This paper, however, follows the same form of welfare representation used by AP. The advantage of the welfare formulation expressed in (9) is that the social value

[14]Specifically, $E[W_K(\kappa(\theta), 0, \theta)(K - \kappa(\theta))] = \mid W_{KK} \mid E[(K - \kappa(\theta))\kappa(\theta)] \left(\frac{E[\kappa(\theta)(\kappa^*(\theta) - \kappa(\theta))]}{E[(\kappa(\theta))^2]} \right)$, where $\phi = \left(\frac{E[\kappa(\theta)(\kappa^*(\theta) - \kappa(\theta))]}{E[(\kappa(\theta))^2]} \right)$.

of information results can be explicitly tied to each of the inefficiencies of the equilibrium allocation, thus making its interpretation more intuitive.

5.1 Social value of public information

The total welfare effect of changing the precision of the noise in the public signal can be written as

$$\frac{d(E[u])}{d\tau_v} = \left(\frac{\partial(E[u])}{\partial\tau^m}\right)_{\gamma^m\ cons.} \frac{d\tau^m}{d\tau_v}. \tag{10}$$

There are two effects on equilibrium welfare. The first is related to how, whilst keeping the weight fixed on the private signal, a change in the precision of the noise in the public signal affects equilibrium welfare. The first effect has the same sign as when public information is exogenous. The second concerns how the overall precision of the endogenous public signal increases as a result of an increase in τ_v (i.e., $0 < \frac{d\tau^m}{d\tau_v}$ from Corollary 2). Consequently, the first effect fully determines the sign of the social value of public information. However, the magnitude of the social value of public information may differ between endogenous and exogenous public information because $\frac{d\tau^m}{d\tau_v} < \kappa_1^2$. If $\mid \kappa_1 \mid \leq 1$ then the magnitude of social value of public information is smaller than $\left(\frac{\partial(E[u])}{\partial\tau^m}\right)_{\gamma^m\ cons.}$. The inequality is reversed if the full information response to fundamentals is sufficiently large.

The main result shows the necessary and sufficient conditions for determining whether increasing τ_v has positive or negative social value.[15]

Proposition 5

Define $\iota = (1-\alpha)(2\alpha - \alpha^* - 1 - 2\phi(1-\alpha^*))$, $\rho = -(1+2\phi)(1-\alpha^*)$, and $\hat{Y} = \frac{-\iota}{\rho}$ if $\rho \neq 0$. It then follows that

$$\frac{d(E[u])}{d\tau_v} \gtreqless 0 \Longleftrightarrow \iota\tau_\xi + \rho\tau^m \lesseqgtr 0, \tag{11}$$

such that

$$sign(\iota) = sign(I-\phi), \quad sign(\rho) = sign(R-\phi), \tag{12}$$

where $I = \frac{2\alpha-\alpha^*-1}{2(1-\alpha^*)}$, and $R = \frac{-1}{2}$.

[15] I assume that the welfare planner cannot change the precision of the ex ante fundamentals, τ_θ.

Proposition 5 shows that the ratio of payoff parameters that determines the social value of public information, $\hat{Y}$, is the same both with exogenous and endogenous public information. Hence, given (10) and Corollary 2, Proposition 5 can be implied from the results of UY. However, since the welfare representation of (9) emphasizes on the inefficiencies of the equilibrium allocation, Proposition 5 allows us to intuitively interpret how the social value of public information depends on the combination of sign, size and relative weight of each of the different components of equilibrium welfare loss (dispersion, volatility, and first order effect in WL) summarized by ι, ρ.

Specifically, increasing τ_v: (1a) decreases dispersion since agents place less weight on the private signal; (2a) reduces agents' forecast error about θ, which decreases volatility, but (2b) also increases the weight that agents place on the public signal, which implies that $K - \kappa(\theta)$ is larger, and hence that volatility increases; (3a) if $\phi \neq 0$ then there is a first order effect in WL. Increasing τ_v lowers an agent's forecast error based on public information, hence K becomes closer to $\kappa(\theta)$, and if $\phi > 0$, the distance between K and $\kappa^*(\theta)$ becomes smaller which is improves welfare; if $\phi < 0$ then increasing τ_v is reduces welfare since the difference between K and $\kappa^*(\theta)$ becomes larger and the first order effect in WL increases.

Combining the effects of increasing τ_v on the three components of WL, I obtain the following results. First, $\rho < 0$ if and only if (2a) and (3a) are jointly beneficial for welfare, and R is the threshold level of the full information inefficiency such that the first order effect in WL offsets the positive welfare effect (2a). Second, $\iota < 0$ if and only if (1a), (2b) and (3a) are beneficial for welfare, and I is the cutoff level of ϕ that balances the combination of (1a) and (2b) with the first order effect in WL.

5.2 Social value of private information

When public information is endogenous, the total welfare effect of changing the precision of the private signal is

$$\frac{d(E[u])}{d\tau_\xi} = \left(\frac{\partial(E[u])}{\partial\tau_\xi}\right)_{\tau^m\, cons.} + \left(\frac{\partial(E[u])}{\partial\tau^m}\right)_{\gamma^m\, cons.} \frac{d\tau^m}{d\tau_\xi}. \tag{13}$$

The social value of private information with an endogenous public signal is the sum of two effects. First, the direct effect that changing τ_ξ has on equilibrium welfare, whilst the precision of the noise in the public signal remains fixed. The sign is the same as when public information is exogenous. Second, there is an indirect effect since $\frac{d\tau^m}{d\tau_\xi} > 0$ (as shown in Corollary 2), and therefore affects total welfare. The sign of this indirect effect is determined by $\left(\frac{\partial(E[u])}{\partial\tau^m}\right)_{\gamma^m\, cons.}$.

Due to the endogenous public signal, the social value of private information is related to whether public information is beneficial or detrimental. Adding the two effects, I notice that endogenous public information may change both the sign and the magnitude of the total welfare effect that results from changing τ_ξ in relation to the direct welfare effect. This augments the social value of private information if the social value of public information is positive, and it decreases the social value of private information if the social value of public information is negative.

The main proposition spells out the necessary and sufficient conditions for equilibrium welfare to increase or decrease with respect to τ_ξ.

Proposition 6

Define $\varphi = -(1-\alpha)^2(1-\alpha+2\phi(1-\alpha^*))$, $\chi = (1-\alpha)((2\alpha^*-\alpha-1)-2\phi(1-\alpha^*))$, *and* $\hat{X} = \frac{-\left(\varphi+\iota\frac{d\tau^m}{d\tau_\xi}\right)}{\chi+\rho\frac{d\tau^m}{d\tau_\xi}}$ *if the denominator is non-zero.*[16] *It then follows that*

$$\frac{d(E[u])}{d\tau_\xi} \gtrless 0 \Longleftrightarrow (\varphi\tau_\xi+\chi\tau^m)+\frac{d\tau^m}{d\tau_\xi}(\iota\tau_\xi+\rho\tau^m) \lesseqgtr 0, \tag{14}$$

such that

$$sign\left(\varphi+\iota\frac{d\tau^m}{d\tau_\xi}\right) = sign(F-\phi), \;\; sign\left(\chi+\rho\frac{d\tau^m}{d\tau_\xi}\right) = sign(C-\phi), \tag{15}$$

where

$$F = \frac{-(1-\alpha)^2+(2\alpha-\alpha^*-1)\frac{d\tau^m}{d\tau_\xi}}{2(1-\alpha^*)((1-\alpha)+\frac{d\tau^m}{d\tau_\xi})}, \textit{ and } C = \frac{(1-\alpha)(2\alpha^*-\alpha-1)-(1-\alpha^*)\frac{d\tau^m}{d\tau_\xi}}{2(1-\alpha^*)((1-\alpha)+\frac{d\tau^m}{d\tau_\xi})}. \tag{16}$$

Proposition 6 shows how the social value of private and public information combine in establishing the sign of the social value of private information. Notice that the ratio of payoff and informational parameters $\hat{X}$ is different from the corresponding one with exogenous public information, $\hat{X}_{exo} = \frac{-\varphi}{\chi}$, since the former depends on the strength of the information externality. Using Proposition 5, (13) and Corollary 2, the result can be implied by the work of UY. As mentioned earlier, the advantage of the formulation of Proposition 6 is that the social value of private information can be explicitly tied to the potential inefficiencies of the equilibrium allocation. The effects of increasing τ_ξ on the components of equilibrium welfare loss are the following.

[16] For completeness, define $\hat{X} = -\infty$ if $\chi+\rho\frac{d\tau^m}{d\tau_\xi} = 0$

Dispersion: (1A) keeping τ^m fixed, dispersion diminishes since information is more accurate but, (1B) it also increases dispersion since agents place more weight on the private signal. Due to endogenous public information, there is also an additional effect (1C) which reduces dispersion since $\frac{d\tau^m}{d\tau_\xi} > 0$ and agents place more weight on the public signal.

Volatility: (2A) when τ^m remains fixed, volatility reduces since agents place less weight on public information. Due to endogenous public information: (2B) volatility is lower since a higher τ^m increases the accuracy of information, but (2C) volatility increases since agents place more weight on public information, thus making overall volatility larger.

First order effect in WL: (3A) if $\phi \neq 0$ and keeping τ^m fixed, there is a reduction in the first order effect in WL if $\phi > 0$ since agents place less weight on the public signal thus reducing $K - \kappa(\theta)$. The opposite occurs if $\phi < 0$. Because of endogenous public information, the raise in τ^m causes an additional effect (3B) that if $\phi > 0$ the first order effect in WL decreases, whereas if $\phi < 0$ then the first order effect increases.

The sign and magnitude of these effects are summarized by $\varphi, \chi, \iota, \rho$ and by a characteristic of the endogenous information structure, which jointly determine the social value of private information. Parameter φ contains effects (1A) and (3A), while parameter χ combines (1B), (2A) and (3A). Hence, $\chi < 0$ and $\varphi < 0$ if and only if the combined effects are beneficial for welfare. The effects of the social value of private information also depend on ι, ρ, which have been described in section 5.1, and are weighted by $\frac{d\tau^m}{d\tau_\xi}$. In addition, F and C correspond to the threshold levels of full information inefficiency, ϕ, such that the first order effects in WL offset the corresponding second order effects in equilibrium welfare loss.

5.3 Combining the social value of public and private information

In symmetric quadratic payoff games of incomplete and exogenous information, UY show how changing the precision of information causes different welfare effects which form eight types of possible games:$-I, -II, -III, -IV, +I, +II, +III, +IV$. Table 1 presents the classification that is obtained with an endogenous information structure. The advantage of the welfare representation used here is that the types of game are defined according to the sign and size of the full information externality (ϕ) in relation to $\alpha - \alpha^*$. Therefore, the type of game is directly related to the different kinds of externalities.

Table 1: Types of games with an endogenous public signal.

α, α^*	ϕ	$\hat{X}, \hat{Y}$	Type of game	Information condition	$\frac{dE[u]}{d\tau_\xi}$	$\frac{dE[u]}{d\tau_v}$
$\alpha > \alpha^*$	$\phi \leq C$	$\hat{X} < \hat{Y} < 0$	$-I$	$\forall\, \tau_\xi, \tau_v$	$-$	$-$
	$C < \phi \leq R$	$0 < \hat{X}$	$+IV$	$\hat{X} < \frac{\tau^m}{\tau_\xi}$	$+$	$-$
				$\frac{\tau^m}{\tau_\xi} < \hat{X}$	$-$	$-$
	$R < \phi < F$	$0 < \hat{X} < \hat{Y}$	$+III$	$\hat{Y} < \frac{\tau^m}{\tau_\xi}$	$+$	$+$
				$\hat{X} < \frac{\tau^m}{\tau_\xi} < \hat{Y}$	$+$	$-$
				$\frac{\tau^m}{\tau_\xi} < \hat{X}$	$-$	$-$
	$F \leq \phi < I$	$\hat{X} \leq 0 < \hat{Y}$	$+II$	$\hat{Y} < \frac{\tau^m}{\tau_\xi}$	$+$	$+$
				$\frac{\tau^m}{\tau_\xi} < \hat{Y}$	$+$	$-$
	$I \leq \phi$	$\hat{X} < \hat{Y} \leq 0$	$+I$	$\forall\, \tau_\xi, \tau_v$	$+$	$+$
$\alpha^* > \alpha$	$\phi \leq I$	$\hat{X} < \hat{Y} \leq 0$	$-I$	$\forall\, \tau_\xi, \tau_v$	$-$	$-$
	$I < \phi \leq F$	$\hat{X} \leq 0 < \hat{Y}$	$-II$	$\hat{Y} < \frac{\tau^m}{\tau_\xi}$	$-$	$-$
				$\frac{\tau^m}{\tau_\xi} < \hat{Y}$	$-$	$+$
	$F < \phi < R$	$0 < \hat{X} < \hat{Y}$	$-III$	$\hat{Y} < \frac{\tau^m}{\tau_\xi}$	$-$	$-$
				$\hat{X} < \frac{\tau^m}{\tau_\xi} < \hat{Y}$	$-$	$+$
				$\frac{\tau^m}{\tau_\xi} < \hat{X}$	$+$	$+$
	$R \leq \phi < C$	$0 < \hat{X}$	$-IV$	$\hat{X} < \frac{\tau^m}{\tau_\xi}$	$-$	$+$
				$\frac{\tau^m}{\tau_\xi} < \hat{X}$	$+$	$+$
	$C \leq \phi$	$\hat{X} < \hat{Y} < 0$	$+I$	$\forall\, \tau_\xi, \tau_v$	$+$	$+$
$\alpha^* = \alpha$	$\phi < R$	$\hat{X} = \hat{Y} < 0$	$-I$	$\forall\, \tau_\xi, \tau_v$	$-$	$-$
	$R < \phi$	$\hat{X} = \hat{Y} < 0$	$+I$	$\forall\, \tau_\xi, \tau_v$	$+$	$+$

Note. The table assumes that R and C cannot be simultaneously equal to zero.

Table 1 shows the following possible types of games (refer to UY for the equivalent definitions with an exogenous information structure):

- Types $+I$ ($-I$): the social value of private and public information are positive (negative) for any precisions τ_ξ, τ_v.

- Types $+II$ ($-II$): i) if $\hat{Y} < \frac{\tau^m}{\tau_\xi}$ then the social value of private information is positive (negative), while social value of public information is positive (negative); if $\frac{\tau^m}{\tau_\xi} < \hat{Y}$ then the social value of private information is positive (negative), while social value of public information is negative (positive).

- Types $+III$ ($-III$): i) if $\hat{Y} < \frac{\tau^m}{\tau_\xi}$ then the social value of public and private information are positive (negative); ii) if $\hat{X} < \frac{\tau^m}{\tau_\xi} < \hat{Y}$ then the social value of private information is positive (negative), while the social value of public information is negative (positive); iii) if $\frac{\tau^m}{\tau_\xi} < \hat{X}$ then the social value of public and private information are negative (positive).

- Types $+IV$ ($-IV$): i) if $\hat{X} < \frac{\tau^m}{\tau_\xi}$ then the social value of private information is positive (negative), while the social value of public information is negative (positive); if $\frac{\tau^m}{\tau_\xi} < \hat{X}$ then the social value of private information is negative (positive), while the social value of public information is negative (positive).

From Table 1, we observe that the interaction of inefficiencies ($\alpha - \alpha^*$, ϕ, and the information externality) determine which of the effects on WL dominate, and these specify the type of game: if $\alpha > \alpha^*$ then the game can be of types $+I, +II, +III, +IV, -I$; if $\alpha^* > \alpha$ then the game can be of types $-I, -II, -III, -IV, +I$; and if $\alpha = \alpha^*$ then the game can only be of types $-I$ and $+I$. In addition, notice that if agents with full information sufficiently under-respond to the fundamental (i.e., ϕ is positive and sufficiently large) then the game is of type $+I$ because the benefits of more precise information through the first order effect in WL are greater than the potential negative second order effects. However, if agents with full information sufficiently over-respond to the fundamental (i.e., as ϕ becomes sufficiently negative) then the benefits of either more precise public or private information disappear and the game is of type $-I$. Specific applications often have restrictions on the payoff parameters and only a few types of games may apply. Proposition 7 explores how the endogenous public signal modifies the cutoffs between and within types of games.

Proposition 7

For a given information structure, endogenous public information modifies the following (in relation to when public information is exogenous):

- *Cutoff between types of games: If $\alpha > \alpha^*$ ($\alpha^* > \alpha$) then there is a larger range of ϕ that satisfy the conditions for types $+III, -I$ ($-III, +I$), and a smaller range of ϕ that satisfy the conditions for types $+II, +IV$ ($-II, -IV$).*

- *Cutoffs within types of games: The range of $\frac{\tau^m}{\tau_\xi}$ that satisfies the conditions for a positive social value of private information is smaller in games of types $+III$ and $+IV$, while it is larger in games of types $-III$ and $-IV$.*

Due to the endogenous public signal, there may be the following differences in the types of games compared to an exogenous information structure: (i) payoff parameters alone do not always determine the type of game since the characteristics of the information structure are significant; (ii) some of the cutoffs between types change (those that depend on the parameters related to the social value of private information); (iii) some of the cutoffs within types are altered (those that depend on $\hat{X}$); (iv) the modifications to the cutoffs depend on the relationship between α and α^*. Due to the endogenous public signal, these potential changes in cutoffs show that the social value of private information might be overturned from positive to negative if $\alpha > \alpha^*$, and from negative to positive if $\alpha^* > \alpha$.

However, there are two scenarios in which the endogenous public signal does not change the sign of the social value of information. The first case corresponds to games with no payoff inefficiencies ($\alpha = \alpha^*$ and $\phi = 0$), which might be described as a game of type $+I$ either with endogenous or exogenous public information. An example is when firms compete in a homogeneous product market and total surplus is the welfare benchmark (described in section 3.1). The second case corresponds to situations where the information externality is not strong enough to overturn the main conclusions of the social value of information. This occurs in the beauty contest (e.g., Morris and Shin 2002, UY). This game's payoff structure satisfies $0 = \alpha^* < \alpha$ and $\phi = 0$. With exogenous public information the game is of types $+I$ and $+II$. With an endogenous public signal, one might wonder if the game could be of type $+III$. However, the payoff parameters of the application ensure that this does not occur and hence cutoffs within and between types of game do not change. In both cases, the endogenous public signal has only a magnitude effect and does not change the sign of the social value of information.

In the next section, I illustrate the intuition of Proposition 7 with two applications where endogenous public information has the opposite effect on the social value of information.

6 Applications: Competition à la Cournot and à la Bertrand with product differentiation

Consider a large market where firms compete (monopolistic competition model) either in quantity (à la Cournot) or price (à la Bertrand) strategies, and sell differentiated products. Each firm is uncertain about a common shock, θ, that affects its demand. To form expectations about θ, a firm has access to both a private signal, as a result of the observation of local market conditions, and to a public signal, which all other firms can access without cost. The public signal is a contemporaneous forecast of the aggregate strategy of all other firms in the market. This aggregate strategy signal contains noise since the aggregation process is imperfect, and due to measurement error. Firms base their strategies on both the private signal and on the public contemporaneous forecast about the aggregate quantity or price, which is endogenous. In contrast, if the information agency tracked and disclosed fundamentals (instead of the market activity) then public information would be exogenous.

Competition à la Cournot A firm faces the linear inverse demand $p_i = \theta - (1-\delta)q_i - \delta Q$, where (p_i, q_i) are price-quantity pairs for firm i, Q is the aggregate quantity produced by all firms, and $\delta \in (0,1)$ can be interpreted as the degree of product differentiation. At the limit, when $\delta \to 0$ firms are isolated monopolies, and if $\delta \to 1$ then there is perfect competition. This payoff structure is from Vives (1990). Firm i's profit is: $\pi_i = \theta q_i - (1-\delta)q_i^2 - \delta q_i Q$. Set $q_i = k_i$ and $Q = K$. Then the profit can be written as

$$U(k, K, \sigma_k, \theta) = \theta k_i - (1-\delta)k_i^2 - \delta k_i K. \tag{17}$$

It can be shown that $W(K, \sigma_k, \theta)$ is equal to total producer surplus, $PS = \theta K - K^2 - (1-\delta)\sigma_k^2$. The characteristics of the payoff structure are as follows: $\phi < 0$ (since $\kappa_1 = \frac{1}{2-\delta} > \kappa_1^* = \frac{1}{2}$); and $\alpha^* = 2\alpha < \alpha < 0$, where $\alpha = \frac{-\delta}{2(1-\delta)}$ (i.e., firms perceive an inefficiently low degree of strategic substitutability in relation to the monopoly benchmark). The next corollary applies the main results to competition à la Cournot.

Corollary 4

i) Equilibrium and efficiency: Firms that compete à la Cournot always give too little weight to private information in relation to the efficient level.
ii) Social value of information. The game is of type $+I$ *if* $-\frac{1}{2} \leq \alpha < 0$*; type* $+II$ *if* $\vartheta \leq \alpha < \frac{-1}{2}$ *such that* $-1 < \vartheta < \frac{-1}{2}$*; and type* $+III$ *if* $\alpha < \vartheta$*, where the cutoff that defines whether the social value of private information is positive in region* $+III$ *satisfies:* $\hat{X}_{exo} < \hat{X}$.

Both with exogenous and endogenous public information, firms that compete à la Cournot underweight private information since $\alpha > \alpha^*$. The size of the difference between the equilibrium and efficient weights to private information is larger with endogenous than with exogenous public information due to the information externality.

In terms of the social value of information, competition à la Cournot can be of three types of game $(+I, +II, +III)$ when public information is either exogenous (as in AP; UY; Bergemann and Morris, 2013; Myatt and Wallace, 2015) or endogenous (Corollary 4). With an exogenous information structure, UY show that the Cournot application can be represented by games of: type $+I$ if $\frac{-1}{2} \leq \alpha < 0$, of type $+II$ if $-1 \leq \alpha < \frac{-1}{2}$, and of type $+III$ if $\alpha < -1$. Correspondingly, the sign of social value of public information is equal to $sign(\frac{\tau_\theta + \tau_\varepsilon}{\tau_\xi} - \hat{Y})$, where $\hat{Y} = \frac{-(2-\delta)(1-2\delta)}{2(1-\delta)^2}$, and the sign of the social value of private information is equal to $sign(\frac{\tau_\theta+\tau_\varepsilon}{\tau_\xi} - \hat{X}_{exo})$, where $\hat{X}_{exo} = \frac{-(2-3\delta)}{(2-\delta)}$.

An explicit comparison of Corollary 4 and the previously known results with exogenous public information (e.g., UY) illustrate the differences in the social value of information due to the endogenous public signal. First, the endogenous public signal modifies the cutoff between games of types $+II$ and $+III$. Specifically, with an endogenous public signal, there exists a degree of strategic complementarity, ϑ, such that $-1 < \vartheta < \frac{-1}{2}$, which delimits whether the game is of type $+II$ or $+III$ (with exogenous public information this cutoff is equal to -1). Hence, due to the endogenous public signal, the range of α that defines the type $+II$ game might decrease, and the one that defines a type $+III$ game increases. Second, in the Cournot application, the endogenous public signal modifies the cutoff within type $+III$ game that determines the social value of private information. For a given τ^m, there is a smaller range of τ_ξ, for which the social value of private information is positive with endogenous public information, than in relation to when public information is exogenous. The strength of the information externality, determined by both payoff and information parameters, determines how large these changes are.

The intuition is that, due to the endogenous public signal, an increase in the precision of the private signal generates an increase in τ^m (even though it is bounded above by $k_1^2\tau_v$). As a result, agents further increase the weight to the public signal, and correspondingly, reduce the weight to the private signal. Given that $\alpha > \alpha^*$, the additional increase in non-fundamental volatility is larger than the reduction in dispersion. Furthermore, since $\phi < 0$ the endogenous public signal generates an additional increase in the first order effect in WL which deteriorates welfare. To conclude, due to the endogenous public signal, the social value of private information may be overturned *from positive to negative*, as reflected in the changes in the cutoffs between and within types of games.

Competition à la Bertrand Firms face the following linear demand $q_i = \theta - p_i + bP$, where (p_i, q_i) are price-quantity pairs for firm i and P is the market's aggregate price. A firm's profit is $\pi_i = p_i q_i - c q_i^2$. Setting $p_i = k_i$ and $P = K$, the firm i's profit can be expressed as

$$U(k, K, \sigma_k, \theta) = (\theta - k_i + bK)k_i - c(\theta - k_i + bK)^2, \tag{18}$$

where $c > 0$ and $0 < b < 1$. Producer surplus is equal to $W(K, \sigma_k, \theta) = (\theta - (1-b)K)K - c(\theta - (1-b)K)^2 - (1+c)\sigma^2$. The payoff structure satisfies: $\phi > 0$ (since $\kappa_1^* = \frac{1+2c(1-b)}{2(1-b)(1+c(1-b))} > \kappa_1 = \frac{(2c+1)}{(2-b)+2c(1-b)} > 0$), and $\alpha^* = \frac{b(1+c(2-b))}{(1+c)} > \alpha = \frac{b(2c+1)}{2(1+c)} > 0$ (i.e., firms perceive a lower degree of strategic complementarity in relation to the monopoly level). Next, I summarize the results for competition à la Bertrand.

Corollary 5

i) Equilibrium and efficiency: When the information externality is small, firms overweight private information in relation to the efficient level. However, when the information externality is large, there exists an $\alpha^o > 0$ such that $\gamma^m(\alpha^o) = \gamma^*(\alpha^o)$ and $sign(\gamma^m - \gamma^*) = sign(\alpha - \alpha^o)$.
ii) Social value of information. Assume that $b < \frac{2(1+c)}{(2c+1)}$. Define the following set $B = \left\{(b,c) \mid \frac{-1+\sqrt{2}}{2} < c, \text{ and } \frac{2c+1}{4c} < b < \frac{2(c+1)}{2c+1}\right\}$, and, for a given information structure, define the non-empty set $A = \left\{(b,c,\tau_\xi,\tau_\theta,\tau_u) \mid \hat{X} < 0 < \hat{X}_{exo}\right\}$ with $A \subset B$. The game is of type $-IV$ if the parameters belong to set $A^c \cap B$, and of type $+I$ if $A \cup B^c$.

When public information is exogenous, firms that compete à la Bertrand always give an inefficiently high weight to private information because $\alpha^* > \alpha$. In contrast, I find that when public information is endogenous, firms that compete à la Bertrand may underweight private information in relation to the efficient level if $\alpha^o > \alpha > 0$, thus overturning one of the main conclusions of models of price setting complementarities (e.g., Hellwig 2005, AP, Colombo et al. 2014). This occurs when the information externality is large.

The results on the social value of information show that the Bertrand application may be represented as game types $+I$ and $-IV$ when public information is either exogenous (as in AP and UY), or endogenous (Corollary 5). With an exogenous information structure, UY show that the Bertrand application is a type $-IV$ game if the parameters belong to set B (defined in Corollary 5); and a type $+I$ game if otherwise. If it is a type $-IV$ game, the sign of social value of private information is equal to $sign(\frac{\tau_\theta+\tau_\varepsilon}{\tau_\xi} - \hat{X}_{exo})$, where $\hat{X}_{exo} = -\frac{((b+2)+2c(3-2b)+4c^2(1-b))}{2(1+c)(1+2c(1-2b))}$.

The changes in the social value of information due to the endogenous public signal are as follows. First, the cutoff between game types $+I$ and $-IV$ changes. Due to the endogenous public signal, there is a smaller region of parameters that defines a type $-IV$ game (region defined by all the elements of set B not contained in A with $A \subset B$) than in relation to when public information is exogenous (region defined by set B). Correspondingly, due to the endogenous public signal, there is a larger region of parameters that defines a type $+I$ game (due to the addition of the region defined by set A). Second, the cutoff within a type $-IV$ game that determines the social value of private information changes due to the endogenous public signal, since in a game of type $-IV$ the cutoff satisfies $\hat{X}_{exo} < \hat{X}$. Hence, for a given τ^m, there is a larger range of precisions of private information for which the social value of private information is positive with endogenous public information than in relation to when public information is exogenous.

The intuition of the results is as follows. More precise public information is always welfare improving because it helps agents reduce the excess equilibrium weight on private information, thus reducing dispersion and the first order effect more than it increases non-fundamental volatility. However, more precise private information may in fact reduce social welfare when public information is either exogenous or endogenous. Due to the endogenous public signal, the welfare effect of increasing the precision of the private signal may be overturned from *negative to positive*. This is because the endogenous public signal fosters greater coordination because higher precision of the private signal increases γ^m, which raises τ^m, and agents give higher weight to the public signal. This results in a further decrease in dispersion, which improves welfare. For a given α, a higher α^* implies that dispersion has a higher weight in WL in relation to volatility. Furthermore, since $\phi > 0$, the endogenous public signal also leads to an additional decrease in the first order effect in WL. To sum up, the overall effect of the endogenous public signal on the social value of private information is positive, as reflected in the changes in cutoffs between and within types. The strength of the effect depends on the characteristics of both the payoff and information structures.

7 Concluding Remarks

I have investigated the effects of an *endogenous public signal* on the use of information and on the social value of information in economies that are characterized by payoff externalities and heterogeneous information about fundamentals. This has been motivated, on the one hand, by the observation that non-price systems for aggregating information are nowadays ubiquitous, and on the other, by the theoretical need to study information externalities that are independent of the market structure.

The endogenous public signal provides information not only about fundamentals, but also about the aggregate action of agents. As a result, agents in equilibrium underweight private information for a larger payoff parameter region in relation to when public information is exogenous. In addition, the welfare planner would like agents to perceive a lower degree of coordination with endogenous, rather than exogenous, public information. The combination of payoff and information externalities characterizes whether agents over-, under-, or equally weight private information in relation to the efficient strategy.

The main contribution in relation to the previous literature is to show that endogenous public information may overturn the sign of the social value of private information. This may be from a positive to a negative sign if agents in equilibrium coordinate more than is implied by the socially optimal exogenous degree of coordination, and the opposite if they coordinate less. In contrast, endogenous public information has only a magnitude effect on the social value of public information. An implication for policy is that information and payoff structures cannot be separated when evaluating the welfare effects of providing more precise public and private information.

A limitation of this paper is that the model considered is static. The main insights should be robust in a dynamic specification of the model. Because the essence of the information externality is the same in either static or dynamic models, where agents respond insufficiently to private information, one might conjecture that the main results would be unaffected by the dynamic element. Nevertheless, a multi-period model would be an interesting extension to develop.

This paper suggests several additional open questions for future research. Future work could explore how robust these results are to more general endogenous information structures, and how different mechanisms for aggregating information affect the use and the social value of public and private information.

Appendix

Proof of Proposition 1

Focusing on linear arbitrary strategies of the form $k(x_i, z) = b' + ax_i + c'E[\theta \mid z]$ and using the definition of the public signal, I obtain: $w = \int k_i + v = b' + a\theta + c'w + v$, whose informational content can be seen to the same as that of $z = a\theta + v$. From the properties of the normal distribution stated in the text, I find that $E[\theta|x_i, z] = \frac{\tau_\xi}{\tau_\xi+\tau(a)}x_i + \frac{\tau(a)}{\tau_\xi+\tau(a)}E[\theta \mid z]$.

Using the maximization condition in the REE definition, I find that the best response is a strategy k', satisfies $E[U_k(k', K, \sigma_k, \theta) \mid x_i, z] = 0$ and hence $k' = E[(1-\alpha)\kappa(\theta) + \alpha K|x_i, w]$. The second order condition is satisfied since $U_{kk} < 0$. Equating coefficients, I obtain: $b = \kappa_0$, $a = \frac{\kappa_1(1-\alpha)\tau_\xi}{(1-\alpha)\tau_\xi+\tau(a)}$ and $c = \frac{\kappa_1\tau(a)}{(1-\alpha)\tau_\xi+\tau(a)}$. Hence, $k(x_i, z) = \kappa_0 + \kappa_1(\gamma x_i + (1-\gamma)E[\theta|z])$, where $\gamma = \frac{(1-\alpha)\tau_\xi}{(1-\alpha)\tau_\xi+\tau_\theta+\kappa_1^2\gamma^2\tau_v}$. The equilibrium weight to private information, γ^m, is then a solution of $\Gamma(\gamma^m) = 0$, where $\Gamma(\gamma) = \gamma^3\kappa_1^2\tau_v + \gamma((1-\alpha)\tau_\xi + \tau_\theta) - (1-\alpha)\tau_\xi$. By the Descartes' Rule of signs, I note that there is only one sign change and, therefore, there is only a positive real root. Checking $\Gamma(-\gamma)$, I notice that there are no sign changes, and therefore, there are no negative real roots. Additionally, $\Gamma(0) = -(1-\alpha)\tau_\xi < 0$ and $\Gamma(1) = \kappa_1^2\tau_v + \tau_\theta > 0$, and, therefore, the positive real root is between 0 and 1. □

Proof of Corollary 1

Differentiating $\Gamma(\gamma^m) = 0$ with respect to the various information and payoff parameters, I obtain that:

$\frac{\partial\gamma^m}{\partial\tau_\xi} = \frac{(1-\alpha)(1-\gamma^m)}{(1-\alpha)\tau_\xi+\tau_\theta+3(\gamma^m)^2\kappa_1^2\tau_v} > 0$; $\frac{\partial\gamma^m}{\partial\tau_\theta} = \frac{-\gamma^m}{(1-\alpha)\tau_\xi+\tau_\theta+3(\gamma^m)^2\kappa_1^2\tau_v} < 0$;

$\frac{\partial\gamma^m}{\partial\tau_v} = \frac{-(\gamma^m)^3\kappa_1^2}{(1-\alpha)\tau_\xi+\tau_\theta+3(\gamma^m)^2\kappa_1^2\tau_v} < 0$; and $\frac{\partial\gamma^m}{\partial\alpha} = \frac{-(1-\gamma^m)\tau_\xi}{(1-\alpha)\tau_\xi+\tau_\theta+3(\gamma^m)^2\kappa_1^2\tau_v} < 0$ since $0 < \gamma^m < 1$. □

Proof of Corollary 2

The proof follows from: $\frac{d\tau^m}{d\tau_\xi} = (2\kappa_1^2\tau_v\gamma^m)\frac{(1-\alpha)(1-\gamma^m)}{(1-\alpha)\tau_\xi+\tau_\theta+3(\gamma^m)^2\kappa_1^2\tau_v} > 0$;
$0 < \frac{d\tau^m}{d\tau_v} = \kappa_1^2\gamma^2(\frac{(1-\alpha)\tau_\xi+\tau_\theta+(\gamma^m)^2\kappa_1^2\tau_v}{(1-\alpha)\tau_\xi+\tau_\theta+3(\gamma^m)^2\kappa_1^2\tau_v}) < \kappa_1^2$; and $0 < \frac{d\tau^m}{d\tau_\theta} = \frac{(1-\alpha)\tau_\xi+\tau^m}{(1-\alpha)\tau_\xi+\tau_\theta+3(\gamma^m)^2\kappa_1^2\tau_v} < 1$. $\square$

Proof of Proposition 2

The welfare planner internalizes all externalities and maximizes ex ante utility, or equivalently minimizes welfare loss. Focusing on a linear arbitrary efficient strategy of the form $k^*(x_i, z) = b^* + a^*x_i + c^*E[\theta \mid z]$. By a similar argument as the one used in Proposition 1, $b^* = \kappa_0^*$ and $k^*(x_i, z) = \kappa_0^* + \kappa_1^*(\gamma x_i + (1-\gamma)E[\theta|z])$, where γ is an arbitrary weight to the private signal. The first order condition satisfies $\frac{dL^*}{d\gamma}|_{\gamma=\gamma^*} = 0$. Hence a candidate efficient strategy with weight to private information γ and precision given by $\tau(\kappa_1^*\gamma) = \tau_\theta + (\kappa_1^*\gamma)^2\tau_v$ solves

$$\frac{\gamma}{\tau_\xi} - \frac{(1-\alpha^*)(1-\gamma)}{\tau(\gamma)} - \frac{(\kappa_1^*)^2\tau_v\gamma(1-\alpha^*)(1-\gamma)^2}{\tau(\gamma)^2} = 0, \tag{A1}$$

where the last term is due to the endogeneity of public information. The previous expression can be rewritten as

$$\gamma = \frac{(1-\alpha^*)\tau_\xi}{(1-\alpha^*)\tau_\xi + \tau_\theta + \kappa_1^{*2}\tau_v\gamma^2 - (1-\alpha^*)(1-\gamma)^2\frac{\tau_v\tau_\xi\kappa_1^{*2}}{(\tau_\theta+(\kappa_1^*)^2\gamma^2\tau_v)}}. \tag{A2}$$

Then, γ^* is a solution of a quintic equation $\psi(\gamma^*) = 0$, where

$$\psi(\gamma) = (\gamma)^5{\tau_v}^2(\kappa_1^*)^4 + 2(\gamma)^3(\kappa_1^*)^2\tau_v\tau_\theta + (\gamma)^2(\kappa_1^*)^2\tau_v\tau_\xi(1-\alpha^*) + \gamma(\tau_\theta^2 + \tau_\xi(1-\alpha^*)(\tau_\theta - \tau_v(\kappa_1^*)^2)) - \tau_\xi\tau_\theta(1-\alpha^*).$$

The SOC guarantees that the denominator of $\gamma^* = \frac{(1-\alpha^*)\tau_\xi}{(1-\alpha^*)\tau_\xi+\tau^*-(1-\alpha^*)(1-\gamma^*)^2\frac{\tau_v\tau_\xi(\kappa_1^*)^2}{(\tau_\theta+(\kappa_1^*)^2\gamma^2\tau_v)}}$ is positive and hence $\gamma^* > 0$. Applying Descartes' Rule of signs to find out the number of real roots. There is only one change in sign, and therefore, this polynomial has only one real positive root. Furthermore, $\psi(0) = -\tau_\xi\tau_\theta(1-\alpha^*) < 0$, while $\psi(1) = {\tau_v}^2(\kappa_1^*)^4 + 2(\kappa_1^*)^2\tau_v\tau_\theta + \tau_\theta^2 > 0$ and the unique positive real root is between 0 and 1. $\square$

Proof of Corollary 3

Differentiating $\psi(\gamma^*) = 0$ with respect to the various information and payoff parameters, I obtain that:
$\frac{\partial\gamma^*}{\partial\tau_\xi} = \frac{\gamma^*(1-\gamma^*)(1-\alpha^*)(\tau_v(\kappa_1^*)^2)}{5(\gamma^*)^4{\tau_v}^2(\kappa_1^*)^4+6(\gamma^*)^2(\kappa_1^*)^2\tau_v\tau_\theta+2\gamma^*(\kappa_1^*)^2\tau_v\tau_\xi(1-\alpha^*)} > 0$;
$\frac{\partial\gamma^*}{\partial\tau_\theta} = \frac{-2\gamma^*\tau^*+\tau_\xi(1-\alpha^*)(1-\gamma^*)}{5(\gamma^*)^4{\tau_v}^2(\kappa_1^*)^4+6(\gamma^*)^2(\kappa_1^*)^2\tau_v\tau_\theta+2\gamma^*(\kappa_1^*)^2\tau_v\tau_\xi(1-\alpha^*)} < 0$;
$\frac{\partial\gamma^*}{\partial\alpha^*} = \frac{-(\gamma^*\tau_\xi\tau_v(\kappa_1^*)^2+\tau_\xi\tau_\theta)(1-\gamma^*)}{5(\gamma^*)^4{\tau_v}^2(\kappa_1^*)^4+6(\gamma^*)^2(\kappa_1^*)^2\tau_v\tau_\theta+2\gamma^*(\kappa_1^*)^2\tau_v\tau_\xi(1-\alpha^*)} < 0$ which follow since $0 < \gamma^* < 1$.

Notice that the sign of $\frac{\partial \gamma^*}{\partial \tau_v} = \kappa_1^2 \gamma^* (\frac{-2(\gamma^*)^2 \tau^* + \tau_\xi (1-\alpha^*)(1-\gamma^*)}{5(\gamma^*)^4 \tau_v{}^2 (\kappa_1^*)^4 + 6(\gamma^*)^2 (\kappa_1^*)^2 \tau_v \tau_\theta + 2\gamma^* (\kappa_1^*)^2 \tau_v \tau_\xi (1-\alpha^*)})$ is ambiguous. $\square$

Proof of Proposition 3

The efficient strategy γ^* satisfies $\frac{dL^*}{d\gamma}|_{\gamma=\gamma^*} = 0$, where $L^*(\gamma)$ is a strictly convex function of γ. Hence $sign\,(\gamma^m - \gamma^*)$ can be found from inspecting the sign of $\frac{dL^*}{d\gamma}\ |_{\gamma=\gamma^m}$. Substituting for the equilibrium strategy in $\frac{dL^*}{d\gamma}\ |_{\gamma=\gamma^m}$, I obtain that

$$sign\left(\frac{dL^*}{d\gamma}\ |_{\gamma=\gamma^m}\right) = sign\,(\gamma^m - \gamma^*) = sign\left(\alpha^* - \alpha - \frac{(1-\alpha^*)(1-\alpha)(\kappa_1^2 \tau_v \tau_\xi)}{((1-\alpha)\tau_\xi + \tau^m)^2}\right), \quad \text{(A3)}$$

which implies that $sign\,(\gamma^m - \gamma^*) = sign\,(\hat{\alpha} - \alpha)$, where $\hat{\alpha} = \alpha^* - \frac{(1-\alpha^*)(1-\alpha)(\kappa_1^2 \tau_v \tau_\xi)}{((1-\alpha)\tau_\xi + \tau^m)^2}$. $\square$

Proof of Proposition 4

Follows immediately from Propositions 2 and 3 since $\kappa(\theta) = \kappa^*(\theta)\ \forall \theta\ and\ \alpha = \hat{\alpha} \Leftrightarrow k(x_i, z) = k^*(x_i, z)$. $\square$

Proof of Proposition 5

Substituting γ^m into (9), I obtain:

$$WL = \frac{|\ W_{\sigma\sigma}\ |\ \kappa_1^2}{2}\left(\frac{(1-\alpha)^2 \tau_\xi + (1-\alpha^*)\tau^m}{((1-\alpha)\tau_\xi + \tau^m)^2} + \frac{2\phi(1-\alpha^*)\tau^m}{(1-\alpha)\tau_\xi + \tau^m}\right). \quad \text{(A4)}$$

Comparative statics of ex ante utility with respect to τ_v are equivalent to the opposite comparative statics of WL. Since $\frac{dWL}{d\tau_v} = \left(\frac{\partial WL}{\partial \tau^m}\right)_{\gamma\ cons.} \frac{d\tau^m}{d\tau_v}$ then

$$\left(\frac{\partial WL}{\partial \tau^m}\right)_{\gamma^m\ cons.} = \frac{|\ W_{\sigma\sigma}\ |\ (\kappa_1)^2}{2}\left(\frac{\iota \tau_\xi + \rho \tau^m}{(\tau_\xi(1-\alpha) + \tau^m)^3}\right), \quad \text{(A5)}$$

where ι and ρ are defined in Proposition 5. Recall from Corollary 2 that $0 < \frac{d\tau^m}{d\tau_v}$. Hence, I find that $\iota \gtrless 0 \Longleftrightarrow I = \frac{2\alpha - \alpha^* - 1}{2(1-\alpha^*)} \gtrless \phi$ and $\rho \gtrless 0 \Longleftrightarrow R = \frac{-1}{2} \gtrless \phi$. $\square$

Proof of Proposition 6

Noting that $\frac{dWL}{d\tau_\xi} = \left(\frac{\partial WL}{\partial \tau_\xi}\right)_{\tau^m\ cons.} + \left(\frac{\partial WL}{\partial \tau^m}\right)_{\gamma\ cons.} \frac{d\tau^m}{d\tau_\xi}$, I obtain:

$$\left(\frac{\partial WL}{\partial \tau_\xi}\right)_{\tau^m\ cons.} = \frac{|\ W_{\sigma\sigma}\ |\ (\kappa_1)^2}{2}\left(\frac{\varphi \tau_\xi + \chi \tau^m}{(\tau_\xi(1-\alpha) + \tau^m)^3}\right), \quad \text{(A6)}$$

where φ and χ are defined in Proposition 6. Now rearranging the condition for the social value of information, I obtain $\frac{d(E[u])}{d\tau_\xi} \gtrless 0 \Longleftrightarrow (\varphi + \iota \frac{d\tau^m}{d\tau_\xi})\tau_\xi + (\chi + \rho \frac{d\tau^m}{d\tau_\xi})\tau^m \lesseqgtr 0$. Furthermore, $(\varphi + \iota \frac{d\tau^m}{d\tau_\xi}) \gtrless 0 \Longleftrightarrow F = \frac{-(1-\alpha)^2 + (2\alpha - \alpha^* - 1)\frac{d\tau^m}{d\tau_\xi}}{2(1-\alpha^*)((1-\alpha) + \frac{d\tau^m}{d\tau_\xi})} \gtrless \phi$, and

$(\chi + \rho \frac{d\tau^m}{d\tau_\xi}) \gtrless 0 \Longleftrightarrow C = \frac{(1-\alpha)(2\alpha^* - \alpha - 1) - (1-\alpha^*)\frac{d\tau^m}{d\tau_\xi}}{2(1-\alpha^*)((1-\alpha) + \frac{d\tau^m}{d\tau_\xi})} \gtrless \phi$. Notice that the denominators of F and C are always strictly positive. $\square$

Proof of Proposition 7

i) Using the definitions of the cutoffs, I can obtain that: If $\alpha > \alpha^*$ then $I > F > R > C$; if $\alpha^* > \alpha$ then $C > R > F > I$; and if $\alpha^* = \alpha$ then $C = R = F = I$. Manipulating the expressions previously defined, we can obtain that $F > F_{exo}$, $C > C_{exo} \Leftrightarrow (\alpha - \alpha^*)\frac{d\tau^m}{d\tau_\xi} > 0$. Hence, $sign(\alpha - \alpha^*) = sign(F - F_{exo}) = sign(C - C_{exo})$, and the result follows.

ii) Games of type $+III$ have: $\iota > 0, \varphi + \iota\frac{d\tau^m}{d\tau_\xi} > 0$ and $\rho < 0, \chi + \rho\frac{d\tau^m}{d\tau_\xi} < 0, \chi < 0$. Hence $\hat{X} = \frac{-\left(\varphi + \iota\frac{d\tau^m}{d\tau_\xi}\right)}{\chi + \rho\frac{d\tau^m}{d\tau_\xi}} > \frac{-\varphi}{\chi} = \hat{X}_{exo}$ since $(\varphi\rho - \iota\chi)\frac{d\tau^m}{d\tau_\xi} > 0$ because $0 < \varphi\rho - \iota\chi$. Similar arguments follow for games of type $+IV$ that have $\iota > 0, \varphi + \iota\frac{d\tau^m}{d\tau_\xi} > 0$ and $\rho > 0, \chi + \rho\frac{d\tau^m}{d\tau_\xi} < 0, \chi < 0$; games of type $-III$ that have $\iota < 0, \varphi + \iota\frac{d\tau^m}{d\tau_\xi} < 0$ and $\rho > 0, \chi + \rho\frac{d\tau^m}{d\tau_\xi} > 0, \chi > 0$; and games of type $-IV$ that have $\iota < 0, \varphi + \iota\frac{d\tau^m}{d\tau_\xi} < 0$ and $\rho < 0, \chi + \rho\frac{d\tau^m}{d\tau_\xi} > 0, \chi > 0$. $\square$

Proof of Corollary 4

i) Follows from Proposition 3 by noting that $\gamma^m < \gamma^*$ will always hold since $\alpha > \alpha^*$.

ii) The parameters which determine the social value of information are as follows: $\rho = -1 < 0$, $\iota = -(1-\alpha)(1+2\alpha)$, $\varphi = -(1-\alpha)(1+\alpha)$, $\chi = -(1-\alpha)^2 < 0$, since $\phi = \frac{\alpha}{1-2\alpha}$ and $\alpha^* = 2\alpha < \alpha < 0$. Note that $R = \frac{-1}{2} < \phi < 0$. We also note that $\hat{X}_{exo} = \frac{-(2-3\delta)}{(2-\delta)}$ and $\hat{Y} = \frac{-(2-\delta)(1-2\delta)}{2(1-\delta)^2}$. By Table 1, note that the game can be of types $+I, +II, +III$. If $-\frac{1}{2} \leq \alpha < 0$ then both $\hat{Y}, \hat{X} < 0$ and the game is of type $+I$; if $\alpha < -1$ then both $\hat{Y}, \hat{X} > 0$ and the game is of type $+III$. Due to the endogenous public signal, the range defined by $-1 \leq \alpha < \frac{-1}{2}$ gets divided into two: (1) if $\hat{X}_{exo} < \hat{X} \leq 0 < \hat{Y}$ then the game is of type $+II$; (2) if $\hat{X}_{exo} \leq 0 < \hat{X} < \hat{Y}$ then the game is of type $+III$. The cutoff between types $+II$ and $+III$ changes with endogenous public information and is given by $sign\left(\varphi + \iota\frac{d\tau^m}{d\tau_\xi}\right) = sign(F - \phi)$, where $F = \frac{-(1-\alpha)^2 - \frac{d\tau^m}{d\tau_\xi}}{2(1-2\alpha)((1-\alpha) + \frac{d\tau^m}{d\tau_\xi})}$. Then $\phi > F \Leftrightarrow \Delta(\alpha) = (1-\alpha^2) + \frac{d\tau^m}{d\tau_\xi}(1+2\alpha) > 0$. Note that $\Delta(-1) < 0$, $\Delta(-\frac{1}{2}) > 0$ and $\Delta(\alpha)$ is a strictly increasing function of α since some regularity conditions regarding $\frac{d\tau^m}{d\tau_\xi}$ are satisfied. Hence, there a exists a unique ϑ such that $\Delta(\vartheta) = 0$, where $-1 < \vartheta < \frac{-1}{2}$. Then, if $\vartheta \leq \alpha < \frac{-1}{2}$ the game is of type $+II$, and if $\alpha < \vartheta$ then the game is of type $+III$. From Proposition 7, the cutoff within type $+III$ game changes due to endogenous public information since in this region$\hat{X}_{exo} < \hat{X}$. $\square$

Proof of Corollary 5

i) Follows from Proposition 3 and by noting that $\alpha^* > \alpha$. When the information externality is large there will exist an $\alpha^o = \alpha^* - \frac{(1-\alpha^*)(1-\alpha)(\kappa_1^2\tau_v\tau_\xi)}{((1-\alpha)\tau_\xi + \tau^m)^2} > 0$ because $\gamma(\alpha)$ and $\gamma^*(\alpha^*)$, respectively, are both strictly decreasing function of α, which range from 1 to 0 as $\alpha \to -\infty$ to $\alpha \to 1$ and $\alpha^* \to 1$, respectively. Using the intermediate value theorem and Proposition 3, there exists a unique intersection between the two curves, α^o, which satisfies $0 < \alpha^o < 1$. Hence, $sign\,(\gamma^m - \gamma^*) = sign(\alpha - \alpha^o)$.

ii) Since $\phi > 0$ and $\alpha^* > \alpha > 0$ then the game has $\rho = -\frac{c(2c+1)b^2-4c(1+c)b+(c+1)(2c+1)}{(2c+1)(1+c)} < 0$, $\iota = -\frac{((2-b)+2c(1-b))((b+1)+c(3-b^2)+2c^2(1-b^2))}{2(1+c)^2(2c+1)} < 0$, $\varphi = -\frac{(b-2-2c+2bc)^2((b+2)+2c(3-2b)+4c^2(1-b))}{8(2c+1)(1+c)^3} < 0$, and $\chi = -\frac{(2c(1-b)+(2-b))^2}{4(1+c)^2(2c+1)}(2c(1-2b)+1)$ which might be positive or negative. $\chi > 0$ if and only if parameters (b, c) belong to set B defined in Corollary 5. With endogenous public information, there exists a non-empty region such that $\chi > 0$ and $\chi + \rho\frac{d\tau^m}{d\tau_\xi} < 0$ if and only if the payoff parameters (b, c) and information parameters $(\tau_\xi, \tau_\theta, \tau_u)$ jointly belong to set A defined in Corollary 5. Hence $A \subset B$. Hence, the game is of type $-IV$ if $B \setminus A$ or, equivalently, if $A^c \cap B$. The game is of type $+I$ if $A \cup B^c$. $\square$

References

[1] Amador, M., & Weill, P.O. (2010). Learning from Prices: Public Communication and Welfare. *Journal of Political Economy,* 118(5), 866-907.

[2] Angeletos, G. M., & Werning, I. (2006). Crises and Prices: Information Aggregation, Multiplicity, and Volatility. *American Economic Review,* 96(5), 1720-1736.

[3] Angeletos, G.M., & Pavan, A. (2007). Efficient Use of Information and Social Value of Information. *Econometrica,* 75(4), 1103-1142.

[4] Angeletos, G.M., & Pavan, A. (2009). Policy with Dispersed Information. *Journal of the European Economic Association,* 7(1), 11-60.

[5] Avdjiev, S., McGuire, P., & Tarashev, N. A. (2012). The Social Value of Policy Signals. *BIS Working Paper, 386.*

[6] Banerjee, A. (1992). A Simple Model of Herd Behavior. *Quarterly Journal of Economics,* 107(3), 797-819.

[7] Bergemann, D., & Morris, S. (2013). Robust Predictions in Games with Incomplete Information. *Econometrica*, 81(4), 1251-1308.

[8] Bikhchandani, S., Hirshleifer, D., & Welch, I. (1992). A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades. *Journal of Political Economy,* 100(5), 992-1026.

[9] Bond, P., & Goldstein, I. (2015). Government Intervention and Information Aggregation by Prices. *The Journal of Finance*, 70(6), 2777-2812.

[10] Bru, L., & Vives, X. (2002). Informational Externalities, Herding, and Incentives. *Journal of Institutional and Theoretical Economics,* 158(1), 91-105.

[11] Choi, H., & Varian, H. (2012). Predicting the Present with Google Trends. *Economic Record*, 88(s1), 2-9.

[12] Clark, R.C., & Polborn, M.K. (2006). Information and Crowding Out Externalities. *Economic Theory*, 27(3): 565-581.

[13] Colombo, L., Femminis, G., & Pavan, A. (2014). Information Acquisition and Welfare. *Review of Economic Studies,* 81, 1438-1483.

[14] Diamond, D.W., & Verrecchia, R.E. (1981). Information Aggregation in a Noisy Rational Expectations Economy. *Journal of Financial Economics,* 9, 221-235.

[15] Einav, L., & Levin, J. (2014). Economics in the Age of Big Data. *Science*, 346(6210), 1243089.

[16] Grossman, S. J., & Stiglitz, J. (1980). On the Impossibility of Informationally Efficient Markets. *American Economic Review,* 70(3), 393–408.

[17] Hauk, E., & Hurkens, S. (2001). Secret information acquisition in Cournot markets. *Economic Theory*, 18(3), 661-681.

[18] Hellwig, M. F. (1980). On the aggregation of information in competitive markets. *Journal of Economic Theory*, 22(3), 477-498.

[19] Hellwig, C. (2005). Heterogeneous Information and the Benefits of Transparency. *UCLA mimeo.*

[20] Klemperer, P., & Meyer, M. (1989). Supply Function Equilibria in Oligopoly under Uncertainty. *Econometrica,* 57(6), 1243–1277.

[21] Kovalenkov, A., & Vives, X. (2014). Competitive rational expectations equilibria without apology. *Journal of Economic Theory*, 149(1), 211-235.

[22] McAfee, A., & Brynjolfsson, E. (2012). Big Data: The Management Revolution. *Harvard Business Review,* 90(10), 61-67.

[23] Morris, S., & Shin, H.S. (2002). Social Value of Public Information. *American Economic Review,* 92, 1521-1534.

[24] Morris, S., & Shin, H.S. (2005). Central Bank Transparency and the Signal Value of Prices. *Brookings Papers on Economic Activity,* 2, 1-43.

[25] Myatt, D. P., & Wallace, C. (2015). Cournot Competition and the Social Value of Information. *Journal of Economic Theory,* 158(Part B), 466–506.

[26] Myatt, D. P., & Wallace, C. (2017). Information Use and Acquisition in Price-Setting Oligopolies. *The Economic Journal*, forthcoming.

[27] Radner, R. (1962). Team Decision Problems. *Annals of Mathematical Statistics,* 33, 857-881.

[28] Shorish, J. (2010). Functional Rational Expectations Equilibria in Market Games. *Economic Theory*, 43, 351-376.

[29] Ui, T., & Yoshizawa, Y. (2015). Characterizing Social Value of Information. *Journal of Economic Theory*, 158(Part B), 507–535.

[30] Vives, X. (1988). Aggregation of Information in Large Cournot Markets. *Econometrica,* 56(4), 851-876.

[31] Vives, X. (1990). Trade Association Disclosure Rules, Incentives to Share Information, and Welfare. *RAND Journal of Economics,* 21(3), 409-430.

[32] Vives, X. (2014). On the Possibility of Informationally Efficient Markets. *Journal of the European Economic Association*, 12(5): 1200-1239.

[33] Vives, X. (2017). Endogenous Public Information and Welfare in Market Games. *The Review of Economic Studies,* 84(2): 935-963.